p. 967 # 37
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10/1/04

Review Surface $z = f(x,y)$, point (x_0, y_0, z_0) on the surface.
The tangent plane is:

$$z - z_0 = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

Ex $z = y\cos(x-y)$ at $(2,2,2)$

$f_x = -y\sin(x-y)$ $\qquad f_x(2,2) = 0$

$f_y = \cos(x-y) + y\sin(x-y)$ $\qquad f_y(2,2) = 1$

$$\boxed{z - 2 = 0(x-2) + (y-2)}$$

Def $L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$

$L(x,y) \approx f(x,y)$ near (x_0,y_0) is <u>linear approximation</u>.

Problem Use lin approx to estimate $(1.98)^3\sqrt{(3.01)^2+(3.97)^2}$

$w = x^3\sqrt{y^2+z^2}$

$$dw = \frac{\partial w}{\partial x}dx + \frac{\partial w}{\partial y}dy + \frac{\partial w}{\partial z}dz$$

$$dw = 3x^2\sqrt{y^2+z^2}\,dx + \frac{x^3 y}{\sqrt{y^2+z^2}}\,dy + \frac{x^3 z}{\sqrt{y^2+z^2}}\,dz$$

plug in $x=2, y=3, z=4$ $\quad dx = -.02$ $dy = .01$ $dz = -.03$

$dw = 12 \cdot 5 \cdot (-.02) + \frac{24}{5} \cdot (.01) + \frac{32}{5}(-.03) =$

$-1.2 + .048 - .192 = -1.344$

$W \approx f(2,3,4) + dw \quad \approx 40 - 1.344 \quad = 38.656$

Actual w $\quad 38.87627624$

$W = f(x_1, x_2, \ldots, x_n)$

$$dw = \frac{\partial f}{\partial x_1} dx_1 + \frac{\partial f}{\partial x_2} dx_2 + \cdots + \frac{\partial f}{\partial x_n} dx_n$$

Linear Approx $\quad f(x_0, y_0, z_0) + dw$

CHAIN RULE

Review $\quad f(g(x))' = f'(g(x)) \cdot g'(x)$

Ex $\quad f(x) = x^3$

$g(x) = \sin(x)$

~~$\sin(x^3)$~~ $' =$

$\sin^3(x)' = 3\sin^2 x \cdot \cos x$

$\sin(x^3)' = \cos(x^3)\, 3x^2$

Example

A partical travels along the ellipse $(2\sin t, 3\cos t)$. The temperature in the xy plane is

$$T(x,y) = \frac{1}{x^2+y^2}$$

What is the rate of change of T w.r.t. t at $t = \pi/3$?

$x = (2\sin t)$ $y = 3\cos t$

$$T(t) = \frac{1}{4\sin^2 t + 9\cos^2 t} \quad \text{could do } \frac{dT}{dt}$$

$$t \xrightarrow{\vec{x}} (x,y) \xrightarrow{T} \mathbb{R}$$
$$\mathbb{R} \longrightarrow \mathbb{R}^2 \longrightarrow \mathbb{R}$$

<u>Theorem</u> (Chain Rule part I)

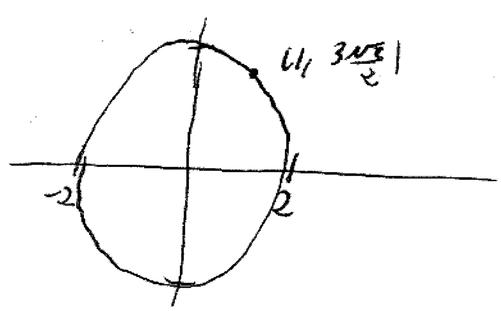
Suppose $z = f(x,y)$ $x = g(t)$ $y = h(t)$. Then

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

<u>Ex</u> $$\frac{dT}{dt} = \frac{-2x}{(x^2+y^2)^2} \cdot 2\cos t + \frac{-2y}{(x^2+y^2)^2} \cdot -3\sin t$$

$$= \frac{-8\sin t\cos t + 18\sin t\cos t}{(4\sin^2 t + 9\cos^2 t)^2} = \frac{10\sin t\cos t}{(4\sin^2 t + 9\cos^2 t)^2}$$

At $t = \pi/3$ $$\frac{\frac{10\sqrt{3}}{4}}{\left(1 + \frac{27}{4}\right)^2} \approx .072$$



Ex $w = xe^{y/z}$ $\quad x = t^2 \quad y = 1-t \quad z = 1+2t$

Find dw/dt

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$

$$= e^{y/z}\cdot 2t + \frac{1}{z}xe^{y/z}(-1) + \frac{-y}{z^2}xe^{y/z}\cdot 2$$

ok to plug in x, y & z now

Example 2

$x = g(s,t) \qquad y = h(s,t)$

$z = f(x,y)$

Thm $$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

Ex $x = s + st^2 \qquad y = \sin(st)$

$z = xy + x^2y^2$

$$\frac{\partial z}{\partial s}$$

$$\frac{\partial z}{\partial t}$$