p.966 # 2, 4, 11, 14, 17, 23, 24, 29, 30

<u>Recall</u>

$f: \mathbb{R}^2 \to \mathbb{R}$ i.e. $f(x,y) = z$

$f: \mathbb{R}^3 \to \mathbb{R}$ i.e. $f(x,y,z) = w$

etc..

<u>partial derivatives</u>

$f_x = \frac{\partial f}{\partial x}, \quad f_y = \frac{\partial f}{\partial y}$ etc.

<u>Ex.</u> $f(x,y) = x \sin(x+2y)$

$$f_x = \sin(x+2y) + x\cos(x+2y)$$
$$f_y = 2x\cos(x+2y)$$
$$f_{xy} = 2\cos(x+2y) - 2x\sin(x+2y)$$
$$f_{yx} = 2\cos(x+2y) - 2x\sin(x+2y)$$

<u>Thm</u> Suppose f_{xy} & f_{yx} are continuous on a disk containing (a,b). Then

$$f_{xy}(a,b) = f_{yx}(a,b)$$

<u>Ex</u> p 958 # 91

$$f(x,y) = \begin{cases} \frac{x^3y - xy^3}{x^2+y^2} & x,y \neq 0,0 \\ 0 & x,y = (0,0) \end{cases}$$

- f is continuous at $(0,0)$
- f_x, f_y, f_{xy}, f_{yx} exist
- $f_{xy}(0,0) = -1 \quad f_{yx}(0,0) = 1$

<u>Problem</u> $z = f(x,y)$. Find tangent plane at point $(x_0, y_0, f(x_0,y_0))$.

<u>Recall</u>

$f_x(x_0,y_0) =$ slope of tangent line to curve of intersection of surface with plane $y = y_0$

$f_y(x_0,y_0) =$ slope of tangent to curve of $\cap$ of surface w/ $x = x_0$ plane.

<u>Know</u> Tang plane is: $A \cdot (x-x_0) + B \cdot (y-y_0) + C(z-z_0) = 0$

or $\quad z - z_0 = a(x-x_0) + b(y-y_0)$

In plane $y = y_0$ we get $\quad z - z_0 = a(x-x_0)$

so $\quad a = f_x(x_0,y_0)$

similarly $\quad b = f_y(x_0,y_0)$

<u>Thm</u> The tangent plane at point (x_0, y_0, z_0) is

$$z - z_0 = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

<u>Example</u> (from exam)

$$f(x,y) = x^2+y^2$$

point $(1,0,1)$

$f_x = 2x \qquad f_x(1,0) = 2$

$f_y = 2y \qquad f_y(1,0) = 0$

$$z-1 = 2(x-1) + 0\cdot(y-0)$$

$$z = 2x-1$$

<u>Ex</u> $f(x,y) = \sqrt{9-x^2-y^2}$ at $(2,1,2)$

<u>Ex</u>

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & (x,y)\neq(0,0) \\ 0 & (x,y)=(0,0) \end{cases}$$

$$f_x = \lim_{h\to 0} \frac{f(x+h,y)-f(x,y)}{h} = \lim_{h\to 0}\left(\frac{(x+h)y}{(x+h)^2+y^2} - \frac{xy}{x^2+y^2}\right)$$

$f_x(0,0) = \lim_{h\to 0} \frac{0}{h^2} = 0 \qquad f_y(0,0) = 0$

tangent plane would be $z=0$.

But $f(x,y) = 1/2$ if $y=x$.

tangent plane should be close to f.