Name: SOLUTIONS

Math 2950- Midterm Exam #1 - September 27, 2004

1. **(10 points)** Find the equation of the plane perpendicular to the line

$$(x, y, z) = (1, -2, 3) + t(1, -2, 1)$$

and passing through the point $(2, 4, -1)$.

We know the normal vector is $(1,-2,1)$

$$\boxed{\langle x-2, y-4, z+1 \rangle \cdot \langle 1,-2,1 \rangle = 0}$$

or expanded out

$$\boxed{x - 2y + z = -7}$$

2. **(15 points)** Let $\vec{u} = (1, -1, 3)$, $\vec{v} = (1, 1, 2)$. Calculate $\vec{u} \cdot \vec{v}$ and $\vec{u} \times \vec{v}$. Then find the angle between $\vec{u}$ and $\vec{v}$.

$$\vec{u} \cdot \vec{v} = 1 - 1 + 6 = \boxed{6}$$

$$\vec{u} \times \vec{v} = (-2-3, \ -2+3, \ 1--1) = (-5, 1, 2)$$

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|} = \frac{6}{\sqrt{11}\sqrt{6}}$$

$$\theta = \cos^{-1}\left(\frac{6}{\sqrt{66}}\right)$$

3. **(10 points)**. Let $\vec{u}$ be a differentiable vector funtion and f a real valued function. Then the chain rule says:

$$\frac{d}{dt}[\vec{u}(f(t))] = ?$$

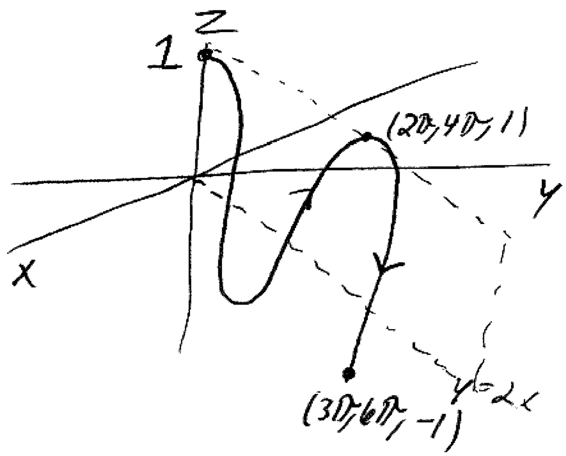
$$\vec{u}'(f(t))\, f'(t)$$

4. **(10 points)** Sketch the space curve

$$\vec{r}(t) = (t, 2t, \cos(t))$$

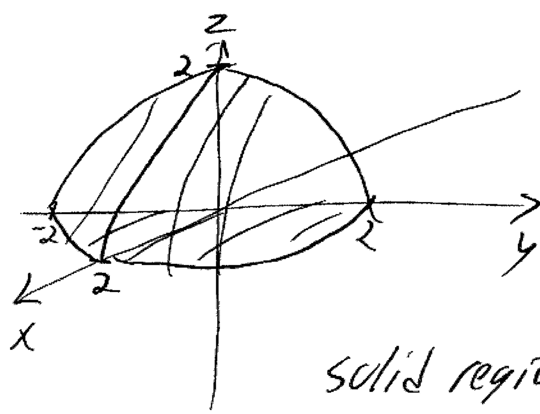
for the interval $0 \le t \le 3\pi$, indicating with an arrow the direction of increasing t.

Notice the curve lies on the plane $y=2x$.



5. **(10 points)** Sketch the region given by the spherical coordinate inequalities:

$$0 \le \phi \le \pi/2$$
$$-\pi/2 \le \theta \le \pi/2$$
$$0 \le \rho \le 2.$$



solid region, ½ of the upper hemisphere.

d. Find the equation for the tangent line to the curve $\vec{r}(t)$ at the point $(1, 0, 1)$.

$$(x, y, z) = (1, 0, 1) + t\,(0, 1, 0)$$

$r'(t) = (-\sin t, \cos t, 0)$

$r'(0) = (0, 1, 0)$

e. Find the equation for the tangent line to the curve $\vec{x}(s)$ at the point $(1, 0, 1)$.

$\vec{x}'(s) = (1, 0, 2s)$

$\vec{x}'(1) = (1, 0, 2)$ $\qquad (x, y, z) = (1, 0, 1) + t\,(1, 0, 2)$

f. Find the equation of the plane passing through the point $(1, 0, 1)$ and containing both tangent lines above. This is called the *tangent plane* to the surface at the point $(1, 0, 1)$.

The normal vector is $\perp$ to both direction vectors for the lines. Thus

$$\vec{n} = (1, 0, 2) \times (0, 1, 0) = (-2, 0, 1)$$

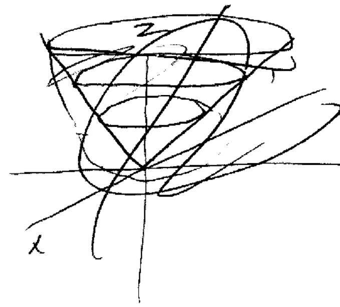
$$(x-1, y, z-1) \cdot (-2, 0, 1) = 0$$

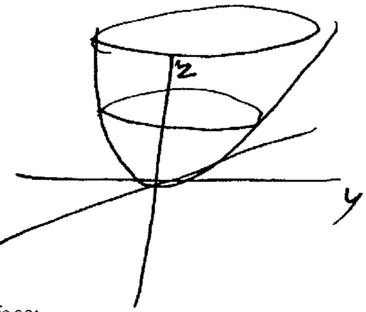
or

$$\boxed{-2x + z = -1}$$

6. **(20 points)**

a. Sketch the surface given by $z = x^2 + y^2$.





b. Verify that the two space curves below both lie on the surface:

$$r(\vec{t}) = (\cos(t), \sin(t), 1) \text{ and } \vec{x}(s) = (s, 0, s^2)$$

$1 = \cos^2 t + \sin^2 t$ so $\vec{r}(t)$ is on the surface

$s^2 = s^2 + 0^2$ so $\vec{x}(s)$ is on the surface

c. Verify that the point $(1,0,1)$ lies on both curves. (i.e. find a t_0 such that $(1,0,1) = \vec{r}(t_0)$ and a s_0 such that $(1,0,1) = \vec{x}(s_0)$.)

$(1,0,1) = \vec{r}(0)$ so $(1,0,1)$ is on $\vec{r}(t)$

$(1,0,1) = \vec{x}(1)$ so $(1,0,1)$ is on $\vec{x}(s)$