Shastri, Anant = Polynomial Representations of Knots
Tohuku Math J. 44 1992 p.11-17

consists of

(1) a *knot-projection*, viz., a generic proper PL-immersion φ:

$$t \mapsto (\alpha(t), \beta(t))$$

of R in R^2 with finitely many crossings (each such crossing being an ordinary double point), and

(2) a set of under(over)-crossing data at each of these crossings.

THEOREM 1. *Every knot-type has a polynomial representation.*

PROOF. Given a knot-type, our task is to find real polynomials $f(t)$, $g(t)$, $h(t)$ such that the map $t \mapsto (f(t), g(t), h(t))$ defines an embedding of C in C^3, and as an embedding of R in R^3, it is in the given knot-type. So, given a knot-type, let $[a, b]$ be the interval in which all the crossings of the knot-projection φ occur. Though strictly not necessary, we may (and are going to) assume that, $\varphi((-\infty, a])$ lies in the third quadrant, and $\varphi([b, \infty))$ lies in the first quadrant. Choose points $a < t_1 < \cdots < t_{2n+1} < b$ in R such that

(i) $\varphi(t_i)$ are smooth points of φ,

(ii) in the interval $[a, b]$ all the crossings are either over-crossings or all are

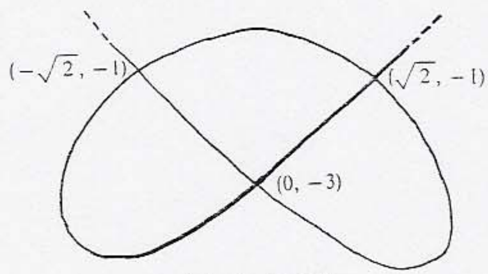
3. The two examples.

1. The trefoil knot:

Set

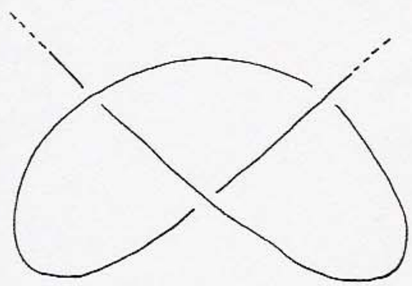
$$f(t) = t^3 - 3t, \quad g(t) = t^4 - 4t^2, \quad h(t) = t^5 - 10t.$$

First, check that the corresponding ring-homomorphism $\varphi : k[X, Y, Z] \to k[t]$ defined by



$(t^3 - 3t, t^4 - 4t^2)$

FIGURE 1.



$(t^3 - 3t, t^4 - 4t^2, t^5 - 10t)$

FIGURE 2.

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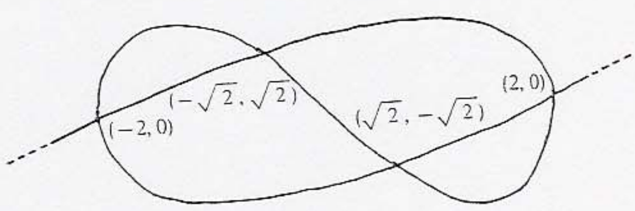
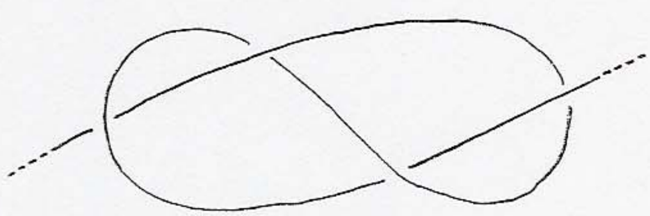
2. The figure eight knot:

Here we define $\psi : C \to C^3$, by taking



$(t^3 - 3t, t^5 - 5t^3 + 4t)$

FIGURE 3.



$(t^3 - 3t, t^5 - 5t^3 + 4t, t^7 - 42t)$

FIGURE 4