

## Math 2950 Sample Exam Problems

1. Sketch in  $R^3$  the two cylindrical solids defined by  $x^2 + z^2 \leq 1$  and  $y^2 + z^2 \leq 1$ . Find the volume of their intersection using a triple integral. Hint: Make the limits on  $z$  as  $-1 \leq z \leq 1$ .
2. A hemispheric bowl of radius 3 inches contains liquid with a depth of two inches. Find the volume of the liquid in the bowl.
3. Consider a lamina that occupies the region  $D$  bounded by the parabola  $x = 1 - y^2$  and the coordinate axes in the first quadrant with density function  $p(x, y) = y$ . Find the mass of the lamina.
4.  $\iiint_E z dV$  where  $E$  is bounded by the planes  $y = 0$ ,  $z = 0$ ,  $x + y = 2$  and the cylinder  $y^2 + z^2 = 1$  in the first octant.
5. Use spherical coordinates to compute the following triple integrals:
  - a.  $\iiint_B \sqrt{x^2 + y^2} dx dy dz$  where  $B$  is the solid ball of radius 1 centered at the origin.
  - b.  $\iiint_B z^2 dx dy dz$  where  $B$  is the first octant region in  $R^3$  between the spheres of radius 2 and of radius 4 centered at the origin.
6. Use the transformation  $x = u^2$ ,  $y = v^2$ ,  $z = w^2$  to find the volume of the region bounded by the (first octant!) surface  $\sqrt{x} + \sqrt{y} + \sqrt{z} = 1$  and the coordinate planes.
7. Find the surface area of the part of the cone  $z^2 = x^2 + y^2$  that lies between  $z = 4$  and  $z = 9$ .
8. p. 1097 1-18, 29-32.
9. Evaluate the line integral  $\int_C \mathbf{F} \cdot d\mathbf{r}$  where  $C$  is given by the vector function  $\mathbf{r}(t)$ :
$$\mathbf{F}(x, y) = (x^2 y^3, -y\sqrt{x}), \mathbf{r}(t) = (t^2, -t^3), 0 \leq t \leq 1.$$
10. Find the work done in moving a particle along the curve  $(x, y, z) = (t, t, t^2)$  from the point  $(0, 0, 0)$  to the point  $(2, 2, 4)$  under the influence of the field  $\mathbf{F}(x, y, z) = (x + y, y, y)$ .