SOLUTIONS

p. 1044

1. $\int_0^{2\pi}\int_0^2 f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$

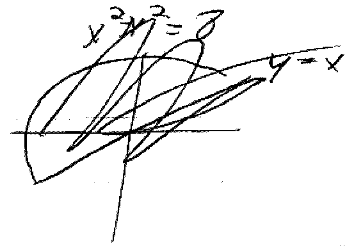
2. $\int_0^2\int_0^{2-x} f(x,y)\,dy\,dx$

3. $\int_{-2}^2\int_x^2 f(x,y)\,dy\,dx$

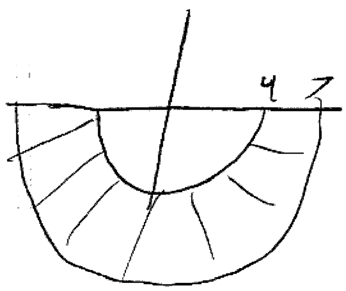
4. $\int_1^3\int_0^{\pi/2} f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$

5. $\int_2\int_0^{2\pi} f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$

6. 



6. $\int_0^{\sqrt{8}}\int_{\pi/4}^{5\pi/4} f(r\cos\theta, r\sin\theta)\,r\,d\theta\,dr$

7. 

$\int_\pi^{2\pi} \frac{r^2}{2}\Big|_4^7 = \int_\pi^{2\pi}\frac{33}{2} = 33\pi/2$

8. $r = 4\cos\theta$

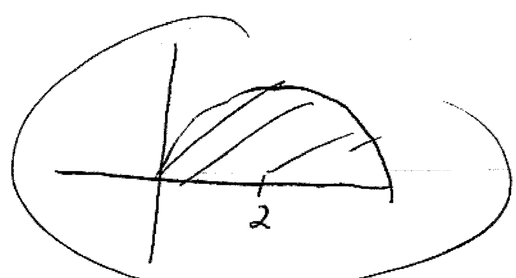
$\sqrt{x^2+y^2} = \frac{4x}{\sqrt{x^2+y^2}}$

$\cos\theta = \frac{x}{r}$

$x^2+y^2 = 4x$

$x^2-4x+y^2=0$

$(x-2)^2+y^2=4$



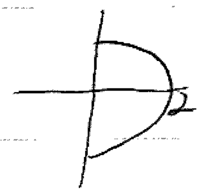
$\int_0^{\pi/2} \frac{r^2}{2}\Big|_0^{4\cos\theta} d\theta = \int_0^{\pi/2} 8\cos^2\theta\,d\theta = 2\pi$

11. $\int_0^{\pi} \int_0^{3} \cos(r^2) r\,dr\,d\theta$

$\int_0^{\pi} \frac{1}{2}\sin(r^2)\Big|_0^3 = \int_0^{\pi} \frac{1}{2}\sin(9)\,d\theta$

$= \frac{\pi}{2}\sin(9)$

13.



$\int_{-\pi/2}^{\pi/2} \int_0^{2} e^{-r^2} r\,dr\,d\theta$

$= \int_{-\pi/2}^{\pi/2} -\frac{1}{2}e^{-r^2}\Big|_0^2 = \int_{-\pi/2}^{\pi/2} -\frac{1}{2}e^{-4} + \frac{1}{2}\,d\theta$

$= \pi\left(-\frac{1}{2}e^{-4} + 1/2\right)$

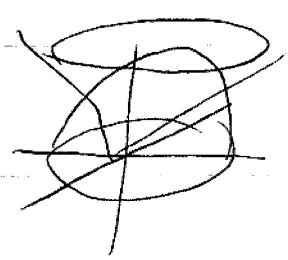
17. $r = \cos 3\theta$

$-\pi/6 \le \theta \le \pi/6$ is one loop

$\int_{-\pi/6}^{\pi/6} \int_0^{\cos 3\theta} r\,dr\,d\theta = \int_{-\pi/6}^{\pi/6} \frac{r^2}{2}\Big|_0^{\cos 3\theta}$

$= \int_{-\pi/6}^{\pi/6} \frac{1}{2}\cos^2 3\theta\,d\theta = \pi/12$

25.



the intersection is

$x^2+y^2+x^2+y^2=1$

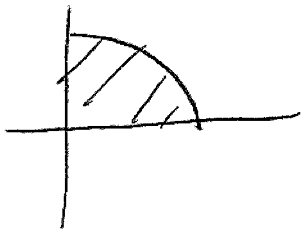
$x^2+y^2=1/2$ so

$\int_0^{2\pi} \int_0^{1/\sqrt{2}} \left(\sqrt{(1-r^2)} - r\right) r\,dr\,d\theta$

$= \int_0^{2\pi} \int_0^{\sqrt{2}/2} r\sqrt{1-r^2} - r^2\,dr\,d\theta$

$= \pi/3(2-\sqrt{2})$

25. 29.



$$\int_0^1\int_0^{\pi/2} e^{r^2} r\,d\theta\,dr$$

$$= \int_0^1 \frac{\pi}{2} r e^{r^2}\,dr$$

$$= \frac{\pi}{4} e^{r^2}\Big|_0^1 = \frac{\pi}{4} e - \pi/4$$

p.1086

29. $$\int_0^2\int_1^4 x^2+4y^2\,dy\,dx = \int_0^2 x^2y+\frac{4}{3}y^3\Big|_1^4\,dx$$

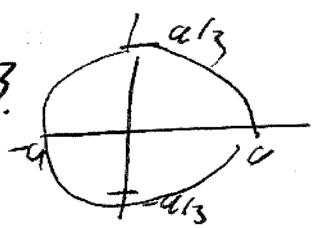
$$= \int_0^2 \left(4x^2+\frac{256}{3}\right)-\left(x^2+\frac{4}{3}\right)dx$$

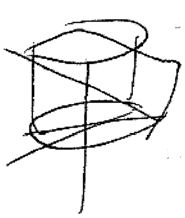
$$= \int_0^2 3x^2+84\,dx = x^3+84x\Big|_0^2$$

$$= 8+168$$

$$= 176$$

33.





$$\int_{-a/3}^{a/3}\int_{-\sqrt{a^2-9y^2}}^{\sqrt{a^2-9y^2}} mx\,dx\,dy = \int_{-a/3}^{a/3} \frac{m}{2}x^2\Big|_{-\sqrt{a^2-9y^2}}^{\sqrt{a^2-9y^2}}\,dy$$

$$= \int_{-a/3}^{a/3} \frac{m}{2}\left(a^2-9y^2+a^2-9y^2\right)dy$$

$$= \int_{-a/3}^{a/3} ma^2-9my^2\,dy$$

$$= ma^2y-3my^3\Big|_{-a/3}^{a/3}$$

33 $= \left(ma^2 \cdot a/3 - 3m\,a^3/27\right) - \left(-ma^3/3 + 3ma^3/27\right)$

$= ma^3/3 - ma^3/9 + ma^3/3 - ma^3/9$

$=$

35. a.

$m = \int_0^1 \int_0^{1-y^2} y\,dx\,dy = \int_0^1 y - y^3\,dy = \frac{y^2}{2} - \frac{y^4}{2}\Big|_0^1 = \boxed{1/4}$

b. $M_x = \frac{\int_0^1 \int_0^{1-y^2} xy\,dx\,dy}{1/4} = \int_0^1 \frac{x^2 y}{2}\Big|_0^{1-y^2} dx\,dy$

$= \int_0^1 \frac{(1-y^2)^2}{2} \cdot y\,dy = \frac{-1/12}{1/4}$

$= \boxed{1/3}$

$M_y = \frac{\int_0^1 \int_0^{1-y^2} y^2\,dx\,dy}{1/4} = \frac{\int_0^1 y^2 - y^4\,dy}{1/4} = \frac{y^3}{3} - \frac{y^5}{5}\Big|_0^1$

$= 2/15 \,/\, 1/4$

$= 8/15$

$\boxed{(\bar{x}, \bar{y}) = (1/3, 8/15)}$

41.

$$\int_{-\pi/2}^{\pi/2}\int_{0}^{3} r\cos\theta\,(r^2)\,r\,dr\,d\theta$$

$$= \int_{-\pi/2}^{\pi/2}\int_{0}^{3} r^4\cos\theta\,dr\,d\theta$$

$$= \int_{-\pi/2}^{\pi/2} \frac{r^5}{5}\Big|_0^3 \cos\theta\,d\theta$$

$$= \frac{243}{5}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta$$

$$= \frac{243}{5}\sin\theta\Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{486}{5}$$