p.1058

2. $z=10-2x-5y$ over region $x^2+y^2 \le 9$

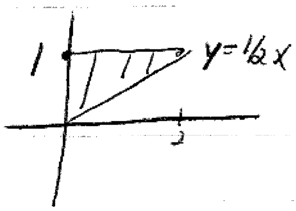
$$\frac{dz}{dx}=-2 \qquad \frac{dz}{dy}=-5$$

$$S.A. = \iint_D \sqrt{1+4+25}\ dA = \sqrt{30}\iint 1\, dA$$

$$= 9\pi\sqrt{30}$$

since $\iint 1 =$ area $D = \pi \cdot 3^2$

4.



$0 \le x \le 2$ OR $0 \le y \le 1$

$\frac{1}{2}x \le y \le 1$ $\quad$ $0 \le x \le 2y$

$$SA = \int_0^2 \int_{\frac{1}{2}x}^1 \sqrt{1+3^2+(4y)^2}\ dy\, dx$$

$$= \int_0^2 \int_{\frac{1}{2}x}^1 \sqrt{16y^2+10}\ dy\, dx$$ oops! I will reverse the order

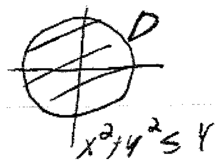
$$\int_0^1 \int_0^{2y} \sqrt{16y^2+10}\ dx\, dy = \int_0^1 2y\sqrt{16y^2+10}\ dy$$

$$= \frac{2}{3}\cdot\frac{1}{16}\left(16y^2+10\right)^{3/2}\Big|_0^1$$

$$= \cancel{\tfrac{1}{24}(75-27)} = 2$$

$$= \frac{1}{24}\left(26^{3/2} - 10^{3/2}\right)$$

6.



$$\iint_D \sqrt{1+4x^2+4y^2}\, dA = \int_0^{2\pi}\int_0^{2} \sqrt{1+4r^2}\, r\, dr\, d\theta$$

$$= \int_0^{2\pi} \frac{2}{3}\cdot\frac{1}{8}\left(1+4r^2\right)^{3/2}\Big|_0^2$$

$$= \int_0^{2\pi} \frac{1}{12}\left(17^{3/2}-1\right)$$

$$= \frac{\pi}{6}\left(17^{3/2}-1\right)$$

9.

$$\iint_{x^2+y^2\le 1} \sqrt{1+x^2+y^2} = \int_0^{2\pi}\int_0^{1} \sqrt{1+r^2}\, r\, dr\, d\theta$$

$$= \int_0^{2\pi} \frac{1}{3}\left(1+r^2\right)^{3/2}\Big|_0^1\, d\theta$$

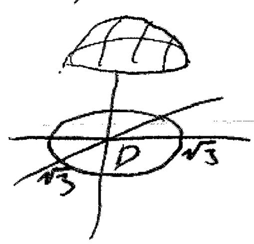
$$= \int_0^{2\pi}\left(\frac{1}{3}\left(2^{3/2}-1\right)\right)$$

$$= \frac{2\pi}{3}\left(2^{3/2}-1\right)$$

10. $z = \sqrt{4-x^2-y^2}$

$$\frac{dz}{dx} = \frac{-x}{\sqrt{4-x^2-y^2}} \qquad \frac{dz}{dy} = \frac{-y}{\sqrt{4-x^2-y^2}}$$

When $z=1$, $x^2+y^2=3$ sc



$$\iint_D \sqrt{1+\frac{x^2}{4-x^2-y^2}+\frac{y^2}{4-x^2-y^2}}\, dA$$

$$= \iint_D \sqrt{\frac{4}{4-x^2-y^2}}\, dA$$ (common denominator!)

$$= \int_0^{2\pi}\int_0^{\sqrt{3}} \frac{2}{\sqrt{4-r^2}}\, r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^{\sqrt{3}} 2\cdot(4-r^2)^{-1/2}\, r\,dr\,d\theta$$

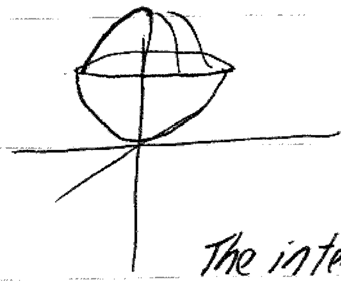
$$= \int_0^{2\pi} -2\,(4-r^2)^{1/2}\Big|_0^{\sqrt{3}}\, d\theta$$

$$= \int_0^{2\pi} -2(1-2) = \int_0^{2\pi} 2\,d\theta = 4\pi$$

12. $x^2+y^2+z^2-4z+4=0\ 4$

$$(x^2)+(y^2)+(z-2)^2=4$$

center $(0,0,2)$ radius 2



The intersection is $z+z^2=4z$

$$z=0,\ z=3$$

When $z=3$, $x^2+y^2=3$ so the region in the xy plane is $x^2+y^2\leq 3$

and $z=\sqrt{4-x^2-y^2}+2$

$$\frac{dz}{dx}=\frac{-x}{\sqrt{4-x^2-y^2}} \qquad \frac{dz}{dy}=\frac{-y}{\sqrt{4-x^2-y^2}}$$

$$\iint_{x^2+y^2\leq 3}\sqrt{1+\frac{x^2}{\sqrt{4-x^2-y^2}}+\frac{y^2}{\sqrt{4-x^2-y^2}}}$$

$$=4\pi \quad (\text{see } \#10!)$$

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#3 $\int_0^1 \int_0^z \int_0^{x+z} 6xz \, dy \, dx \, dz = \int_0^1 \int_0^z 6x^2 z + 6xz^2 \, dx \, dz$

$= \int_0^1 2x^3 z + 3x^2 z^2 \Big|_0^z \, dz$

$= \int_0^1 2z^4 + 3z^4 = z^5 \Big|_0^1 = 1$

4. $\int_0^1 \int_x^{2x} \int_0^y 2xyz \, dz \, dy \, dx = \int_0^1 \int_x^{2x} xyz^2 \Big|_0^y \, dy \, dv$

$= \int_0^1 \int_x^{2x} xy^3 \, dy \, dx = \int_0^1 \frac{1}{4} xy^4 \Big|_x^{2x}$

$= \int_0^1 \frac{1}{4} x \cdot 16x^4 - \frac{1}{4} x^5$

$= \int_0^1 \frac{15}{4} x^5 = \frac{15}{24} x^6 \Big|_0^1 =$

$15/24$

7. $\int_0^2 \int_0^{\sqrt{4-y^2}} \int_0^y 2x \, dz \, dx \, dy = \int_0^2 \int_0^{\sqrt{4-y^2}} 2xy \, dx \, dy$

$= \int_0^2 x^2 y \Big|_0^{\sqrt{4-y^2}} dy$

$= \int_0^2 (4-y^2) y \, dy = 2y^2 - \cancel{y^3/3}\, y^4/4 \Big|_0^2$

$= 8 - \cancel{8/3}\, 4 = \cancel{16/3}$

4

8. $\int_0^1 \int_0^x \int_x^{2x} yz\cos(x^5)\,dz\,dy\,dx$

$= \int_0^1 \int_0^x \frac{yz^2}{2}\cos(x^5)\Big|_{z=x}^{z=2x}\,dy\,dx$

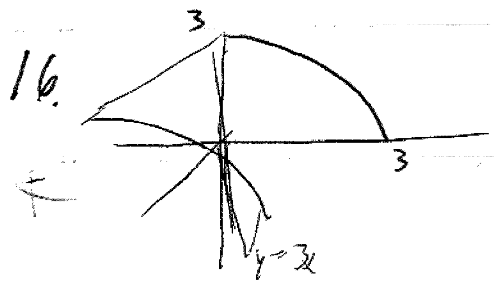
$= \int_0^1 \int_0^x \frac{3}{2} y x^2 \cos(x^5)\,dy\,dx$

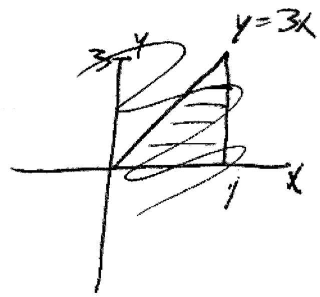
$= \int_0^1 \frac{3}{4} y^2 x^2 \cos(x^5)\Big|_{y=0}^{x} = \int_0^1 \frac{3}{4} x^4 \cos(x^5)\,dx$

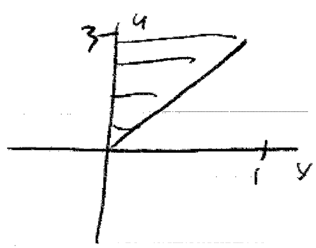
$= \frac{3}{20}\sin(x^5)\Big|_0^1$

$= 3/20 \sin 1$

16.







$\int_0^1 \int_{3x}^3 \int_0^{\sqrt{9-y^2}} z\,dz\,dy\,dx$

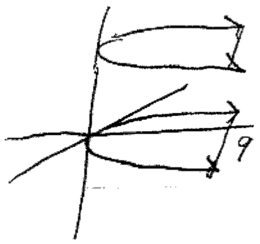
$= \int_0^1 \int_{3x}^3 \frac{z^2}{2}\Big|_0^{\sqrt{9-y^2}}\,dy\,dx$

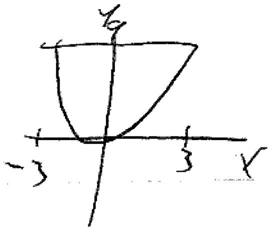
$= \int_0^1 \int_{3x}^3 \frac{1}{2}(9-y^2)\,dy\,dx$

$= \int_0^1 \frac{9}{2}y - \frac{1}{6}y^3 \Big|_{3x}^3\,dx$

$= \int_0^1 \left(\frac{27}{2} - \frac{27}{6} - \frac{27}{2}x + \frac{27}{6}x^3\right)dx$

$= \frac{27}{2}x - \frac{27}{6}x - \frac{27}{4}x^2 + \frac{27}{24}x^4 \Big|_0^1 = \frac{27}{8}$

18. 

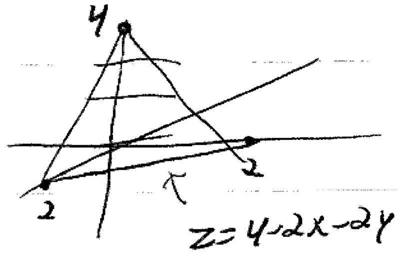


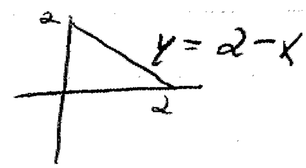
$$-3 \le x \le 3 \qquad x^2 \le y \le 9 \qquad 0 \le z \le 4$$

$$V = \int_0^4 \int_{-3}^{3} \int_{x^2}^{9} dy\,dx\,dz = \int_0^4 \int_{-3}^{3} 9 - x^2\,dx\,dz = \int_0^4 9x - \frac{x^3}{3}\Big|_{-3}^{3} dz$$

$$= \int_0^4 36\,dz = 144$$

p. 1066

10. 

In xy plane 

$$\int_0^2 \int_0^{2-x} \int_0^{4-2x-2y} y\,dz\,dy\,dx$$

$$= \int_0^2 \int_0^{2-x} 4y - 2xy - 2y^2\,dy\,dx$$

$$= \int_0^2 \left(2y^2 - xy^2 - \tfrac{2}{3}y^3\right)\Big|_0^{2-x} dx$$

$$= \int_0^2 2\cdot(2-x)^2 - x(2-x)^2 - \tfrac{2}{3}(2-x)^3$$

$$\cancel{= \int_0^2 (2-x)^3 - \tfrac{2}{3}(2-x^3)\,dx}$$

$$\cancel{= \int_0^2 \tfrac{1}{3}(2-x^3)\,dx = \tfrac{2}{3}x - \tfrac{x^4}{12}\Big|_0^2}$$

$$\cancel{= \tfrac{4}{3} - \tfrac{16}{12} = 0}$$

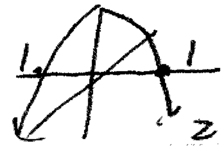
$$\int_0^2 \tfrac{1}{3}(2-x)^3\,dx$$

$$= -\tfrac{1}{12}(2-x)^4\Big|_0^2$$

$$= -\tfrac{1}{12}(0-16)$$

$$= 4/3$$

13



$z=1-y^2$

$-1 \le y \le 1$

$-1 \le x \le 1$

$0 \le z \le 1-y^2$

$\int$

$$\int_{-1}^{1}\int_{-1}^{1}\int_{0}^{(1-y^2)} x^2 e^y \, dz\, dy\, dx$$

$$= \int_{-1}^{1}\int_{-1}^{1} x^2(1-y^2)e^y \, dy\, dx$$

$$= \cancel{\int_{-1}^{1}\int_{-1}^{1}} = \underbrace{\int_{-1}^{1} x^2 dx}_{2/3} \int_{-1}^{1} (1-y^2)e^y$$

Answer $= \frac{2}{3}\int_{-1}^{1}(e^y - y^2e^y)dy$

$\int y^2 e^y dy \qquad u=y^2 \quad dv=e^y$

$du=2y \quad v=e^y \qquad \int u\,dv = uv-\int v\,du$

$= y^2e^y - 2\int ye^y$

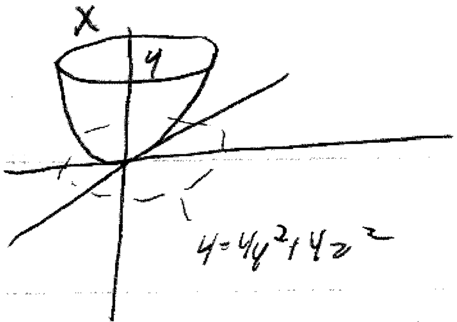
$= y^2e^y - 2(ye^y - e^y) = y^2e^y - 2ye^y + 2e^y$

$$= \frac{2}{3}\left(e^y - y^2e^y + 2ye^y - 2e^y\right)_{-1}^{1}$$

$$= \frac{2}{3}(e-e+2e-2e) - \frac{2}{3}\left(\frac{1}{e} - \frac{1}{e} - \frac{2}{e} - \frac{2}{e}\right)$$

$$= \frac{2}{3}\cdot\frac{4}{e} = 8/3e$$

15.



Use polar $y = r\cos\theta$

$z = r\sin\theta$

$x = x$

so $4r^2 \le x \le 4$

$0 \le r \le 1 \quad 0 \le \theta \le 2\pi$

$$\int_0^{2\pi}\int_0^1\int_{4r^2}^4 x r \, dx\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^1 \frac{r}{2}x^2\Big|_{4r^2}^4 \, dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^1 \frac{r}{2}(16-16r^4)\,dr\,d\theta$$

$$= \int_0^{2\pi} 4r^2 - \frac{8}{6}r^6\Big|_0^1 d\theta = \int_0^{2\pi}\frac{16}{6}\,d\theta = \frac{16}{3}\pi$$

20. As in #15 use polar.

$$\int_0^{2\pi}\int_0^4\int_{r^2}^{16} r \, dx\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^4 r(16-r^2)\,dr\,d\theta = \int_0^{2\pi} 8r^2 - \frac{r^4}{4}\Big|_0^4$$

$$= 2\pi((8\cdot 16) - 64$$

$$= \cancel{256\pi}$$

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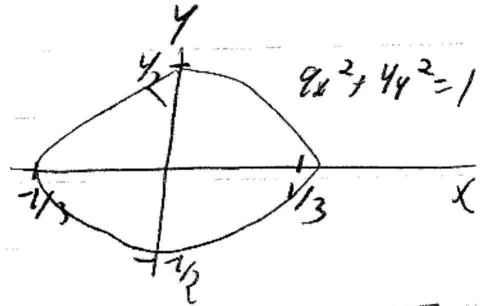
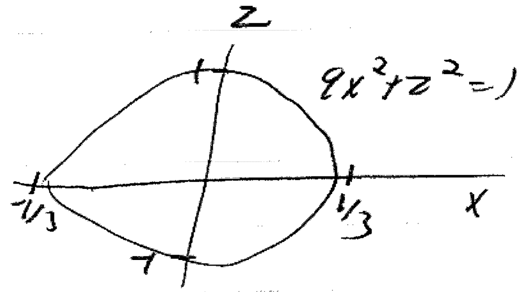
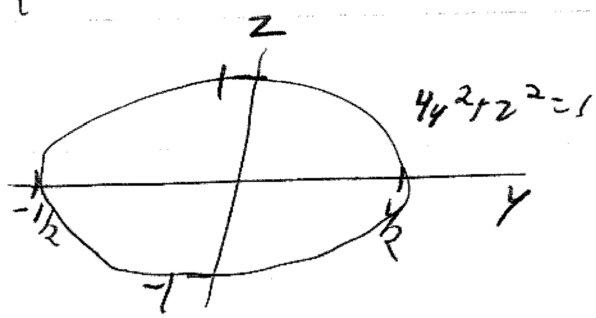
29, 32

29. In back

30.







$$\int_{-1/3}^{1/3}\int_{-\sqrt{1/4-9/4x^2}}^{\sqrt{1/4-9/4x^2}}\int_{-\sqrt{1-9x^2-4y^2}}^{\sqrt{1-9x^2-4y^2}} f\,dz\,dy\,dx$$

$$\int_{-1/2}^{1/2}\int_{-\sqrt{1/9-4/9y^2}}^{\sqrt{1/9-4/9y^2}}\int_{-\sqrt{1-9x^2-4y^2}}^{\sqrt{1-9x^2-4y^2}} f\,dz\,dx\,dy$$

$$\int_{-1/3}^{1/3}\int_{-\sqrt{1-9x^2}}^{\sqrt{1-9x^2}}\int_{-\sqrt{1/4-9/4x^2-1/4z^2}}^{\sqrt{1/4-9/4x^2-1/4z^2}} f\,dy\,dz\,dx$$

$$\int_{-1}^{1}\int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}}\int_{-\sqrt{1/4-9/4x^2-1/4z^2}}^{\sqrt{1/4-9x^2-1/4z^2}} f\,dz\,dy\,dx$$

! 2 more !