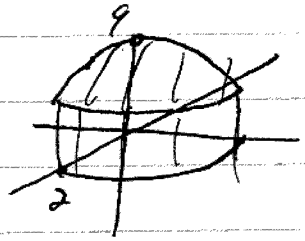
p.1073

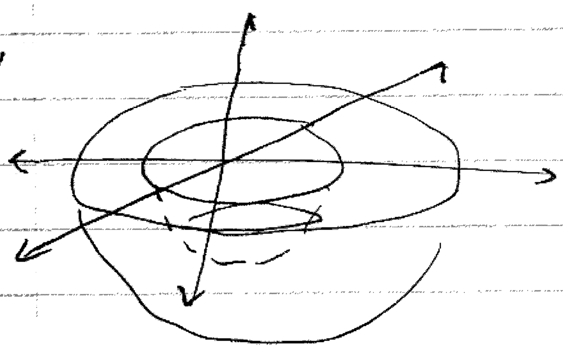
2.



$$\int_0^{\pi/2}\int_0^{2}\int_0^{9-r^2} r\,dz\,dr\,d\theta = \int_0^{\pi/2}\int_0^{2} 9r-r^3\,dr\,d\theta = \int_0^{\pi/2} \frac{9}{2}r^2-\frac{1}{4}r^4\Big|_0^2\,d\theta$$

$$= \int_0^{\pi/2} 18-4\,d\theta = 7\pi$$

4



region inside 2 hemispheres

$$\int_0^{2\pi}\int_{\pi/2}^{\pi}\int_1^{2} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta = \int_0^{2\pi}\int_{\pi/2}^{\pi} \frac{\rho^3}{3}\sin\phi\Big|_1^2\,d\phi\,d\theta$$

$$= \int_0^{2\pi}\int_{\pi/2}^{\pi} \frac{7}{3}\sin\phi\,d\phi\,d\theta$$

$$= \int_0^{2\pi} -\frac{7}{3}\cos\phi\Big|_{\pi/2}^{\pi}\,d\theta$$

$$= \int_0^{2\pi} \frac{7}{3}-0\,d\theta = 14\pi/3$$

12. $$\int_0^1 \int_0^{2\pi} \int_{\sqrt{4-r^2}}^{\sqrt{4-r^2}} r \, dz \, d\theta \, dr$$

$$= \int_0^1 \int_0^{2\pi} 2r\sqrt{4-r^2} \, d\theta \, dr = \int_0^1 4\pi r\sqrt{4-r^2} \, dr$$

$$= -2\pi \tfrac{2}{3} (4-r^2)^{3/2} \Big|_0^1$$

$$= -2\pi \tfrac{2}{3} \left(3^{3/2} - 8\right)$$

$$= \frac{4\pi}{3}\left(8 - 3\sqrt{3}\right)$$

17. $$\int_0^1 \int_0^{2\pi} \int_0^{\pi} \rho^2 \cdot \rho^2 \sin\phi \, d\phi \, d\theta \, d\rho$$

$$= \int_0^1 \int_0^{2\pi} -\rho^4 \cos\phi \Big|_0^{\pi} = \int_0^1 \int_0^{2\pi} 2\rho^4 \, d\theta \, d\rho$$

$$= \int_0^1 4\pi \rho^4 \, d\rho = \cancel{\frac{\pi}{5}}$$

$$= \frac{4\pi}{5}$$

19. $$\int_0^{\pi/2} \int_0^{\pi/2} \int_1^2 \rho\cos\phi \cdot \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \int_0^{\pi/2} \int_0^{\pi/2} \frac{\rho^4}{4} \cos\phi \sin\phi \Big|_1^2$$

$$= \int_0^{\pi/2} \int_0^{\pi/2} \frac{15}{4} \sin\phi \cos\phi \, d\phi \, d\theta = \int_0^{\pi/2} \frac{15}{8} \sin^2\phi \Big|_0^{\pi/2} d\theta$$

$$= \frac{15}{8} \cdot \frac{\pi}{2} = \frac{15\pi}{16}$$

22. $$\int_0^{2\pi}\int_0^{\pi/3}\int_0^{4} \rho\sin\phi\cos\theta\,\rho\sin\phi\sin\theta\,\rho\cos\phi\;\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$= \int_0^{2\pi} \rho^4\,d\rho \cdot \int_0^{\pi/3} \sin^3\phi\cos\phi\,d\phi \underbrace{\int_0^{2\pi} \sin\theta\cos\theta\,d\theta}_{=\frac{\sin^2}{2}\Big|_0^{2\pi}=0}$$

This answer is 0

35.

$0 \le \rho \le 3$

$0 \le \phi \le \pi/2$

$0 \le \theta \le 2\pi$

$$\int_0^{2\pi}\int_0^{\pi/2}\int_0^{3} \rho\cos\phi\cdot\rho\cdot\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$= \int_0^{2\pi} d\theta \int_0^{\pi/2} \sin\phi\cos\phi\,d\phi \cdot \int_0^{3} \rho^4\,d\rho$$

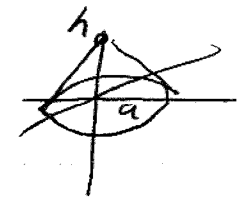
$$= 2\pi \cdot \frac{\sin^2\phi}{2}\Big|_0^{\pi/2} \cdot \frac{\rho^5}{5}\Big|_0^{3}$$

$$= 2\pi\cdot\frac{1}{2}\cdot\frac{243}{5} = 243\pi/5$$

USUAL CONE IS $z=\sqrt{x^2+y^2}$

Instead use

$$z = h - \sqrt{\frac{h^2}{a^2}x^2+\frac{h^2}{a^2}y^2}$$



$$\text{VOL} = \int_0^{2\pi}\int_0^{a}\int_0^{h-\sqrt{\frac{h^2}{a^2}(x^2+y^2)}} r\,dz\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^{a} hr - r\sqrt{\frac{h^2}{a^2}r^2}\,dr\,d\theta \qquad = \frac{r^2h}{a}$$

$$= \int_0^{2\pi} \frac{h}{2}r^2 - \frac{1}{3}r^3\frac{h}{a}\Big|_0^a$$

$$= \int_0^{2\pi} \frac{ha^2}{2} - \frac{1}{3}a^3\frac{h}{a} = \int_0^{2\pi}\frac{1}{6}a^2h$$

$$= \frac{\pi a^2 h}{3}$$

p.1084 #2,4,5,7,8,9,10,13,14

2. $J = \begin{vmatrix} 2u & -2v \\ 2u & 2v \end{vmatrix} = 8uv$

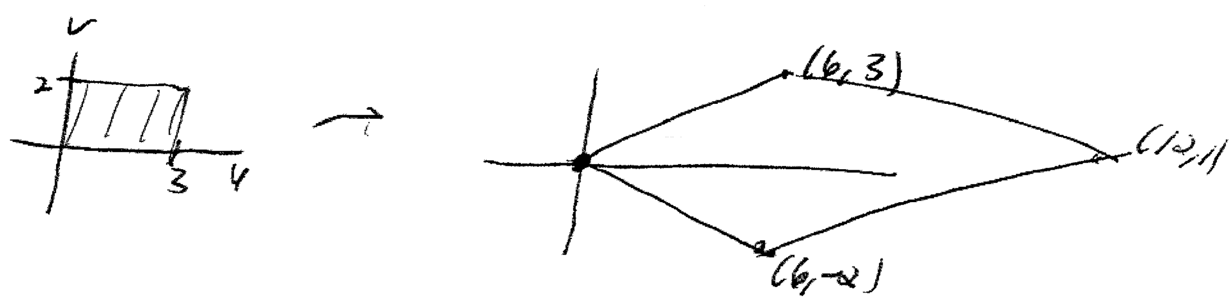
4. $J = \begin{vmatrix} \sin\beta & 2\cos\beta \\ \cos\beta & -2\sin\beta \end{vmatrix} = -2\sin^2\beta - 2\cos^2\beta$

$= -2$

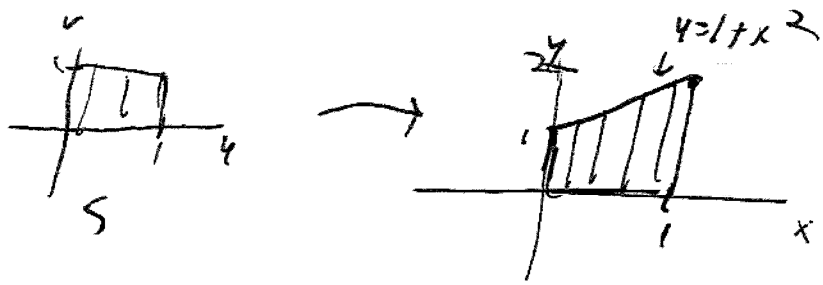
5. $J = \begin{vmatrix} v & u & 0 \\ 0 & w & v \\ w & 0 & u \end{vmatrix} = v\cdot(wu) - u(-vw) + 0$

$= uvw + uvw = 2uvw$

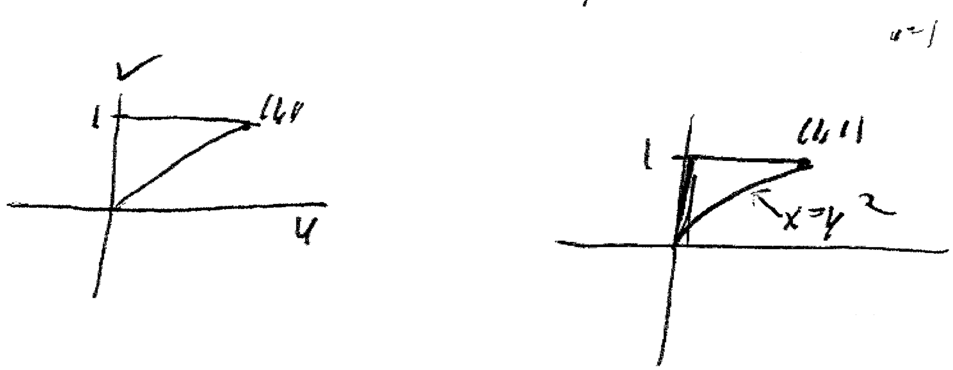
7.



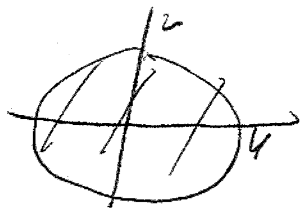
8.

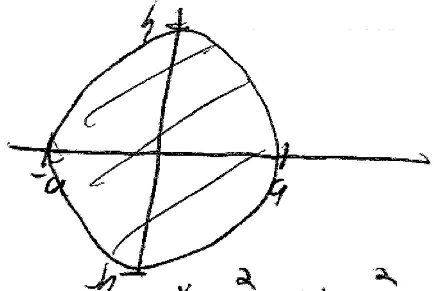


9.



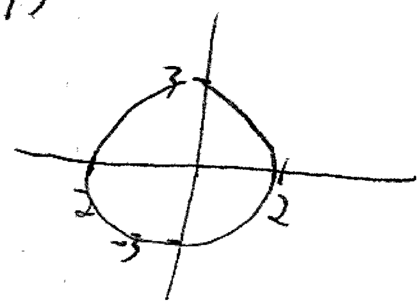
10.





$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 \le 1$

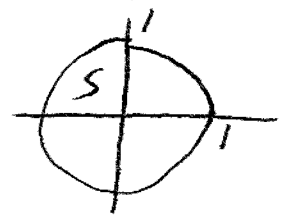
13



$9x^2+4y^2=36$ $\quad$ $x=2u$, $y=3v$ $\Rightarrow$ $36u^2+36v^2=36$

$u^2+v^2=1$

$J = \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} = 6$



$\iint_S (2u)^2 \cdot 6\,du\,dv$

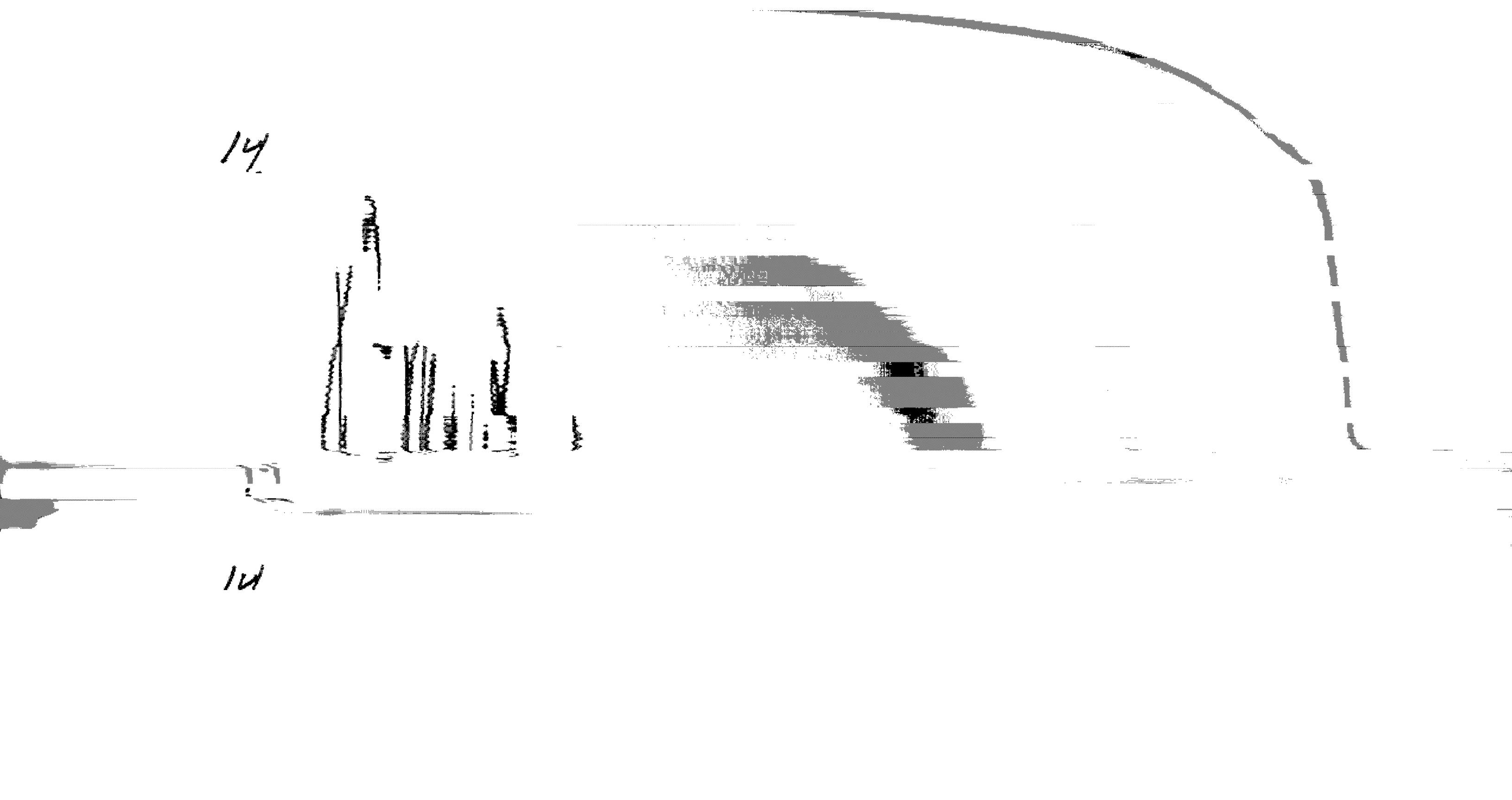
$= \iint_S 24u^2\,du\,dv$ use polar

$\int_0^{2\pi}\int_0^1 24 r^2 \cos^2\theta \; r\,dr\,d\theta$

$= 24\int_0^{2\pi} \cos^2\theta\,d\theta \cdot \int_0^1 r^3\,dr$

$= 24 \cdot \pi \cdot 1/4 = 6\pi$

14



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