SOLUTIONS

p. 1117

1. $\int_C \nabla f \cdot dr = f(\text{end}) - f(\text{start}) = 40$

2. $\int_C \nabla f \cdot dr = f(2,2) - f(1,0) = 9 - 3 = 6$

3. $\frac{\partial Q}{\partial x} = 5 = \frac{\partial P}{\partial y}$ yes!

$f = 3x^2 + 5xy + h(y)$ $f = 5xy + 2y^2 + g(x)$

$f(x,y) = 3x^2 + 2y^2 + 5xy$

5. $\frac{\partial Q}{\partial x} = ye^x$ $\frac{\partial P}{\partial y} = xe^y$ Not conservative

11. $\vec{F}(x,y) = \langle 2xy, x^2 \rangle$ $\frac{\partial Q}{\partial x} = 2x = \frac{\partial P}{\partial y}$. Thus $\vec{F}$ is conservative so line integrals are ind. of path. The potential function is

$f(x,y) = x^2 y$ so

$\int F \cdot dr = \int \nabla f \cdot dr = f(3,2) - f(1,2) = 18 - 2 = 16$

12. $f = yx + g(y)$ $f = xy + y^2 + h(x)$

$\boxed{f(x,y) = y^2 + xy}$

$\int_C F \cdot dr = f(2,1) - f(0,1) = 3 - 1 = 2$

13. $f(x,y) = \frac{1}{4}x^4y^4$

$\int F\cdot dr = f(1,2) - f(0,1) = 4 - 0 = 4$

23. No, clearly $\oint_C F\cdot dr$ is not zero for C a circle around the origin.

p. 1117

15. $f(x,y,z) = xyz + z^2$

$\int F\cdot dr = f(4,6,3) - f(1,0,-2) = 81 - 4 = 77$

16. $f(x,y,z) = x^2z + y^2x + z^3$

$\int F\cdot dr = f(1,2,1) - f(0,1,-1) = 1 + 4 + 1 - (-1) = 7$

17. $f(x,y,z) = xy^2\cos z$

$\int_C F\cdot dr = f(\pi^2, 0, \pi) - f(0,0,0) = 0 - 0 = 0$

19. $\frac{\partial Q}{\partial x} = \sec^2 y \quad \frac{\partial P}{\partial y} = \sec^2 y$

Thus $\vec{F}$ is conservative.

$f(x,y) = x\tan y$ is a potential function

$\int_C F\cdot dr = f(2, \pi/4) - f(1,0) = 2 - 0 = 2$

20. $\frac{\partial Q}{\partial x} = -e^{-x}$ $\frac{\partial P}{\partial y} = e^{-x}$

so $\vec{F}$ is conservative.

$f(x,y) = x + ye^{-x}$ is a potential

$\int_C F \cdot dr = f(1,2) - f(0,1) = 1 + \frac{2}{e} - 1 = 2/e$

27 If $\vec{F}$ is conservative then $P = \frac{\partial f}{\partial x}$ $Q = \frac{\partial f}{\partial y}$ $R = \frac{\partial f}{\partial z}$

for some potential function $f(x,y,z)$.

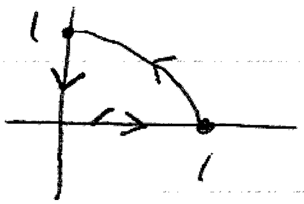
$\frac{\partial P}{\partial y} = f_{xy}$ $\frac{\partial Q}{\partial x} = f_{yx}$ these are $=$ by Clairaut's Thm

Similarly for the other 2 inequalities.

28. $\frac{\partial R}{\partial x} = yz$ $\frac{\partial P}{\partial z} = 0$ so not conservative!

p. 1125

4. Line integral




$C_1 : (0, 1-t) \quad 0 \le t \le 1$

$C_2 : (t, 0) \quad 0 \le t \le 1$

$C_3 : (-t, 1-t^2) \quad -1 \le t \le 0$

$$\oint_C x\,dx + y\,dy = \int_0^1 0 + (1-t)(-1)\,dt + \int_0^1 t\,dt + 0 + \int_{-1}^0 -t(-1)\,dt + (1-t^2)(-2t)\,dt$$

$$= \int_0^1 t-1\,dt + \int_0^1 t\,dt + \int_{-1}^0 t + 2t^3 - 2t$$

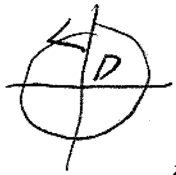
$$= \frac{t^2}{2} - t\Big|_0^1 + \frac{t^2}{2}\Big|_0^1 + \frac{-t^2}{2} + \frac{t^4}{2}\Big|_{-1}^0$$

$$= -1/2 + 1/2 - \left(-\frac{1}{2} + \frac{1}{2}\right) = 0$$

<u>Green's Thm</u>

$$\oint_C F\cdot dr = \iint_D \frac{\partial Q}{\partial y} - \frac{\partial P}{\partial x} = \iint_D 0\,dA = 0$$

11.

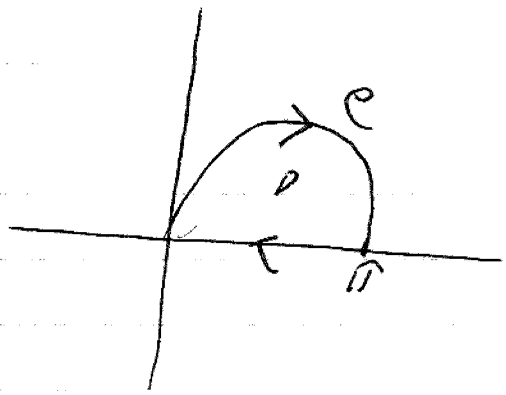



$$\oint_C F\cdot dr = \iint_D -3x^2-3y^2\,dA$$

$$= \int_0^{2\pi}\int_0^2 -3r^2\, r\,dr\,d\theta$$

$$= \int_0^{2\pi} -\frac{3}{4}r^4\Big|_0^2 = -24\pi$$

13.




so Green's Thm applies to $-C$ since $C$ has region on right.

$$\int_C F\cdot dr = -\int_{-C} F\cdot dr$$

$$= -\iint_D 2x-3y^2\,dA$$

$$= -\int_0^{\pi}\int_0^{\sin x} 2x-3y^2\,dy\,dx$$

$$= -\int_0^{2\pi} 2xy-y^3\Big|_0^{\sin x}\,dx$$

$$= -\int_0^{2\pi} 2x\sin x-\sin^3 x\,dx$$

$$\cancel{= -\left[-2x\cos x+2\sin x+\tfrac{1}{3}(2+\sin^2 x)\cos x\right]_0^{2\pi}}$$

$$= -\left(\cancel{\left[-4\pi+\tfrac{2}{3}\right]-\left[\tfrac{2}{3}\right]}\right) =$$

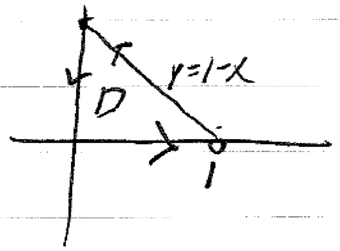
$$= \int_0^{2\pi} \sin^3 x-2x\sin x$$

$$= -\frac{1}{3}(2+\sin^2 x)\cos x+2x\cos x-2\sin x\Big|_0^{2\pi}$$

$$= \left(-\frac{2}{3}\cdot(-1)-2\pi\right)-\left(-\frac{2}{3}\right)$$

$$= 4/3-2\pi$$

17.




$$\oint_C F\cdot dr$$

$$= \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$$

$$= \int_0^1 \int_0^{1-x} y^2 - x \, dy\, dx$$

$$= \int_0^1 \frac{1}{3} y^3 - xy \Big|_0^{1-x} dx$$

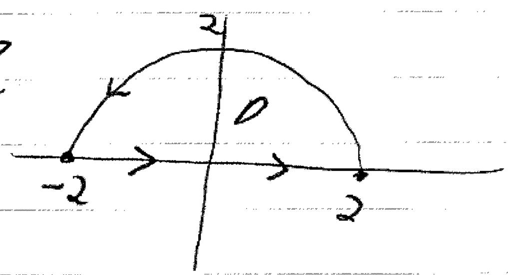
$$= \int_0^1 \frac{1}{3}(1-x)^3 - x + x^2 \, dx$$

$$= -\frac{1}{12}(1-x)^4 - \frac{x^2}{2} + \frac{x^3}{3} \Big|_0^1$$

$$= -\frac{1}{2} + \frac{1}{3} - -\frac{1}{12} = -\frac{6}{12} + \frac{4}{12} + \frac{1}{3}$$

$$= -\frac{1}{12}$$

18.




$$W = \iint_D 3x^2 + 3y^2 - 0 \, dA = \int_0^{2\pi} \int_0^2 3r^2 \, r \, dr \, d\theta$$

$$= \int_0^{2\pi} \frac{3}{4} r^4 \Big|_0^2$$

$$= 24\pi$$