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2. $\text{curl } F = (xz^2 - xy^2, x^2y - yz^2, y^2z - x^2z)$

$\text{div } F = (2xyz + 2xyz + 2xyz) = 6xyz$

5. $\text{curl } F = (0,0,0)$ $\quad \text{div } F = e^x \sin y - e^x \sin y + 1 = 1$

12. a. NA b. NA c. scalar d. vector
e. scalar f. NA g. scalar h. NA
i. vector j. NA k. NA l. scalar

13. yes! $\text{curl} = (0,0,0)$
$f(x,y,z) = xyz + C$

19. If $\text{curl } G = (xy^2, yz^2, zx^2)$ then
$\text{div curl } G = (y^2 + z^2 + x^2)$
but div curl G is always 0 so no!

20. No same as 19
$\text{div curl } G$ would be $xz \neq 0$

32. Thm Every continuous $f(x,y,z)$ is the divergence of some vector field

Pf. Define $g(x,y,z) = \int_0^x f(t,y,z)\,dt$

By FTOC $\frac{\partial g}{\partial x} = f(x,y,z)$

Let $\vec{G}(x,y,z) = \langle g(x,y,z), 0, 0 \rangle$

$\operatorname{div} \vec{G} = \frac{\partial g}{\partial x} + 0 + 0 = f(x,y,z)$ //

#21 Let $\vec{F}(x,y,z) = (f(x), g(y), h(z))$

$\operatorname{curl} \vec{F} = \left(\frac{\partial h}{\partial y} - \frac{\partial g}{\partial z}, \frac{\partial f}{\partial z} - \frac{\partial h}{\partial x}, \frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right)$

$= (0,0,0)$ so $\vec{F}$ is irrotational

22. Let $\vec{F}(x,y,z) = (f(y,z), g(x,z), h(x,y))$

$\operatorname{div} \vec{F} = \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z} \right) = 0+0+0 = 0$

so $\vec{F}$ is incompressible

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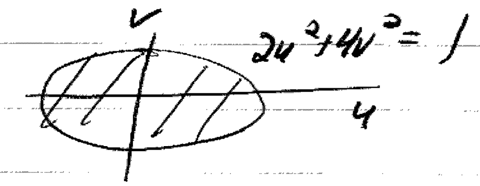
11. IV 12. V 13. I 14. III 15. II 16. IV

17. $\vec{r}(s,t) = \langle 1,2,-3\rangle + s\langle 1,1,-1\rangle + t\langle 1,-1,1\rangle$

18. $z = -\sqrt{1-2x^2-4y^2}$ SC

$\vec{r}(u,v) = \langle u, v, -\sqrt{1-u^2-4v^2}\rangle$

For $u,v \in$ 

22. ~~$\vec{r}(\phi,\theta) = \langle$~~

$z=2$ gives $\rho\cos\phi = 2 \Rightarrow 4\cos\phi = 2 \Rightarrow \phi = \pi/3$

$z=-2$ " " $\phi = 2\pi/3$

SC $\vec{r}(\phi,\theta) = \langle 4\sin\phi\cos\theta,\ 4\sin\phi\sin\theta,\ 4\cos\phi\rangle$

$0 \le \theta \le 2\pi$

$\pi/3 \le \phi \le 2\pi/3$

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31. $r_u = (1, 6u, 1) \to e$

$r_v = (1, u, -1)$ point $(2,3,0)$ is $u=1$ $v=1$

$(x,y,z) = (2,3,0) + s(1,6,1) + t(1,0,-1)$

32. $r_u = (2u, 0, v)$ at $(1,1)$ $\vec{r}(1,1) = (1,1,1)$

$\to = (2,0,1)$

$r_v = (0, 2v, u)$

$\to = (0,2,1)$

$(x,y,z) = (1,1,1) + s(2,0,1) + t(0,2,1)$

33. $r_u = (2u, 2\sin v, \cos v)$ $r_u(1,0) = (2,0,1)$

$r_v = (u, 2u\cos v, -u\sin v)$ $r_v(1,0) = (1,2,0)$

$r(1,0) = (1,0,1)$

$(x,y,z) = (1,0,1) + s(2,0,1) + t(1,2,0)$

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$$SA = \iint_D \sqrt{1+x^2+y^2}\, dA$$

$$= \int_0^{2\pi}\int_0^1 \sqrt{1+r^2}\; r\,dr\,d\theta$$

$$= \int_0^{2\pi} \frac{1}{3}(1+r^2)^{3/2}\Big|_0^1 \, d\theta$$

$$= \frac{2\pi}{3}\left(2^{3/2}-1\right)$$

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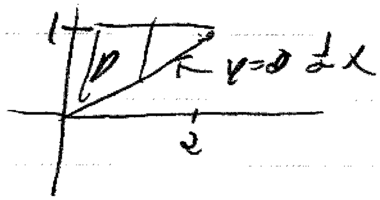

$$SA = \iint_D \sqrt{1+3^2+(4x)^2} = \iint_D \sqrt{10+16y^2}$$

$$= \int_0^1 \int_0^{2y} \sqrt{10+16y^2}\, dx\, dy$$

$$= \int_0^1 2y\sqrt{10+16y^2}\, dy$$

$$= \frac{2}{16\cdot 3}\left(10+16y^2\right)^{3/2}\Big|_0^1$$

$$= \frac{1}{24}\left(26^{3/2}-10^{3/2}\right)$$

45.

$r_u = \langle v, 1, 1 \rangle$

$r_v = \langle u, 1, -1 \rangle$ $\qquad |r_u \times r_v| = |\langle -2, u+v, v-u \rangle|$

$$= \sqrt{4 + u^2 + 2uv + v^2 + v^2 + u^2 - 2uv}$$

$$= \sqrt{4 + 2u^2 + 2v^2}$$

$$SA = \iint_D \sqrt{4 + 2u^2 + 2v^2}$$

$$= \int_0^{2\pi} \int_0^1 \sqrt{4 + 2r^2} \; r \, dr \, d\theta$$

$$= \int_0^{2\pi} \frac{3}{4} (4 + r^2)^{2/3} \Big|_0^1$$

$$= \frac{2\pi \cdot 3}{4} \left(5^{2/3} - 4^{2/3} \right)$$

$$= \int_0^{2\pi} \frac{1}{6} (4 + 2r^2)^{3/2} \Big|_0^1$$

$$= \frac{\pi}{3} \left(6^{3/2} - 4^{3/2} \right)$$