p.1006 4, 8, 13, 18, 38, 39  p.1011 4, 14, 26, 35, 48, 64

4. $\nabla f = (4, 6)$  $\nabla g = (2x, 2y)$

$4 = 2x\lambda$

$6 = 2y\lambda$

$x^2 + y^2 = 13$

$\frac{2}{x} = \frac{3}{y}$

$y = \frac{3}{2}x$

$x^2 + \frac{9}{4}x^2 = 13$

$\frac{13}{4}x^2 = 13$

$x = \pm 2 \quad y = \pm 3$

$x = 2 \quad y = 3 \quad \lambda = 1$

$x = -2 \quad y = -3 \quad \lambda = -1$

$\boxed{\begin{array}{l} f(2,3) = 26 \text{ global max} \\ f(-2,-3) = -26 \text{ global min} \end{array}}$

8. $\nabla f = (8, 0, -4)$  $\nabla(g) = (2x, 20y, 2z)$

$8 = 2\lambda x$  $x^2 + 10y^2 + z^2 = 5$

$0 = 20\lambda y$

$-4 = 2\lambda z$

$\lambda = 0$ not allowed so $y = 0$

$\lambda = 4/x$

$\lambda = -2/z$ $\Rightarrow$ $-\frac{2}{z} = \frac{4}{x}$  $x = -2z$

$4z^2 + z^2 = 5$

$z = 1, x = -2$

$z = -1, x = 2$

$(-2, 0, 1)$ $f = -20$ min

$(2, 0, -1)$ $f = 20$ max

13. $\nabla f = (1,1,1,1)$ $\quad \nabla g = (2x, 2y, 2z, 2t)$

$$\begin{aligned} 1 &= 2\lambda x \\ 1 &= 2\lambda y \\ 1 &= 2\lambda z \\ 1 &= 2\lambda t \end{aligned} \quad \Rightarrow \quad x = y = z = t$$

but $x^2 + y^2 + z^2 + t^2 = 1$

so $4x^2 = 1$

$x = \pm 1/2$

$(1/2, 1/2, 1/2, 1/2) \rightarrow f = 2$ max

$(-1/2, -1/2, -1/2, -1/2)$ $\quad f = -2$ min

18. ~~$\nabla f = (4x, 6y,$~~ $\quad \nabla f = (4x-4, 6y)$

$\nabla g = 2x, 2y$

- critical points for $f$ is $(1, 0)$
- on Boundary $x^2+y^2=16$ we use Lagrange multipliers.

$4x - 4 = 2\lambda x$

$6y = 2\lambda y \qquad y = 0 \quad x = \pm 4$

$y \neq 0 \Rightarrow \lambda = 3 \Rightarrow x = -2$

$x = -2 \rightarrow y = \sqrt{12}$

| pt | f(x,y) |
|---|---|
| (4,0) | 11 |
| (-4,0) | 43 |
| $(-2,\sqrt{12})$ | 47 ← global max |
| (1,0) | -7 ← Global min |

38 Maximize $V = lwh$ subject to:

$$2wh + 2hl + 2wl = 1500, \text{ ie } wh + wl + hl = 750$$
$$4h + 4w + 4l = 200 \qquad h + w + l = 50$$

$\nabla V = (wh, lh, lw)$ $\nabla g_1 = (w+h, h+l, w+l)$ $\nabla g_2 = (1,1,1)$

1.) $wh = \lambda(w+h) + \mu$
2.) $lh = \lambda(h+l) + \mu$
3.) $lw = \lambda(w+l) + \mu$
4.) $wh + wl + hl = 750$
5.) $h + w + l = 50$ 5 equations 5 unknowns

(1)-(2) gives $h(w-l) = \lambda(w-l)$
$w = l$ or $h = \lambda$
(2)-(3) gives $l(h-w) = \lambda(h-w)$ $h = w$ or $l = \lambda$
(3)-3 gives $w(h-l) = \lambda(h-l)$ $h = l$ or $\lambda = w$

~~Suppose $h = \lambda$. Then $wh = wh + h^2 + \mu \Rightarrow h^2 = -\mu$~~

Suppose $w = l$. Then $h + 2w = 50$
$h = 50 - 2w$

#4 $\Rightarrow$ $w(50-2w) + w^2 + w(50-2w) = 750$
$-3w^2 + 100w = 750$
$3w^2 - 100w + 750 = 0$
$$w = \frac{100 \pm \sqrt{1000}}{6} = \frac{100 \pm 10\sqrt{10}}{6}$$
$$= \frac{50 \pm 5\sqrt{10}}{3}$$

$W = \frac{50+5\sqrt{10}}{3} \approx 21.937$ $\quad l = 21.937$

$h = 50 - 2w = 6.1257$ $\quad V = 2947.9$

$W = \frac{50-5\sqrt{10}}{3} \approx 11.396$ $\quad l = 11.396$ $\quad h = 27.208$ $\quad V = 3533.5$

Now suppose $w \neq l$ so $h = \lambda$

#1 gives $\lambda w = \lambda(w+\lambda) + \mu$

$\lambda^2 = -\mu \qquad \mu = -\lambda^2 = -h^2$

#2 $\quad l \cdot \lambda = \lambda(\lambda + l) - \lambda^2$

$l \cdot \lambda = \lambda^2 + \lambda l - \lambda^2$

#3 $\quad lw = h(w+l) - h^2$

$lw = hw + lh - h^2$

Notice it is totally symmetric, assume $h = w$ or $h = l$ gives same answer. So the last case is

$w = l = h = 50/3$ but this doesn't satisfy eq. 4

Max volume 3533.5 $\quad 11.396 \times 11.396 \times 27.208$

Min vol 2947.9 $\quad 21.937 \times 21.937 \times 6.125$

39

Max-minimize $D^2 = \cancel{\phantom{x}}\ x^2+y^2+z^2$

Constraints: $x+y+2z=2$

$z-x^2-y^2=0$

$\nabla f = (2x, 2y, 2z)$

$\nabla g_1 = (1,1,2)$

$\nabla g_2 = (-2x, -2y, 1)$

$\cancel{2x + \lambda = 2x\mu}$

$2x = \lambda + -2x\mu$

$2y = \lambda - 2y\mu$

$2z = 2\lambda + \mu$

$x+y+2z=2$

$z-x^2-y^2=0$

$2x+2x\mu = 2y+2y\mu$

$2x(1+\mu) = 2y(1+\mu)$ so $x=y$ or $\mu=-1$

Case 1 $x=y$ so $2x+2z=2$

$x = 1-z$

$z = 1-x$

$1-x-x^2-x^2=0$

$2x^2+x-1=0$ $x = 1/2$ or $x=-1$

$(1/2, 1/2, 1/2)$ or $(-1,-1,2)$

34. Case 2 $\mu = -1$

$2x = \lambda + 2x$

$2y = \lambda + 2y$ $\lambda = 0$ not allowed!

$4x$

So $(1/2, 1/2, 1/2) \Rightarrow D^2 = 3/4$

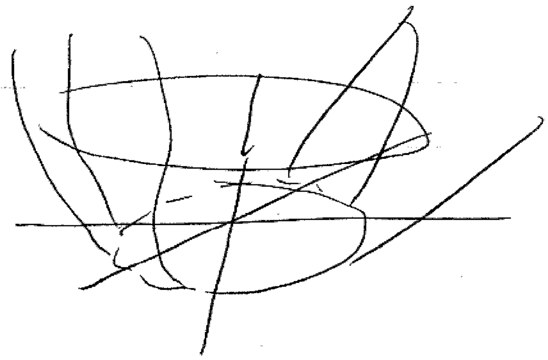
$(-1, -1, 2)$ $D^2 = 6$

$(-1, -1, 2)$ farthest

$(1/2, 1/2, 1/2)$ closest

p1011

4.



Domain is
$x^2+y^2 \geq 1$
top half of hyperboloid

19. $f_x = 12x^2 - y^2$ $f_y = -2xy$

$f_{xx} = 24x$ $f_{xy} = -2y$ $f_{yy} = -2x$

26. $z - e^x \cos y = 0$

$\nabla = (-e^x \cos y, e^x \sin y, 1)$

$\nabla(0,0,1) = (-1, 0, 1)$

a. $(-1,0,1) \cdot (x, y, z-1) = 0$

b. $(0,0,1) + t(-1,0,1) = (x, y, z)$

35. $\dfrac{\partial w}{\partial t} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial t} + \dfrac{\partial w}{\partial z}\dfrac{\partial z}{\partial t}$

$= \dfrac{1}{2\sqrt{x}} \cdot 2e^t + \dfrac{2y}{z}(3t^2+4) - \dfrac{y^2}{z^2} \cdot 2t$

48 $\nabla f = (yze^{xy}, xze^{xy}, e^{xy})$

direction is $\nabla f(0,1,2) = (2,0,1)$

max rate $= \sqrt{5}$

64. maximize $V = lwh$ subject to

$l + 2w + 2h = 108$

$\nabla V = (wh, lh, lw)$ $\quad \nabla g = (1,2,2)$

$wh = \lambda$

$lh = 2\lambda$

$lw = 2\lambda$ $\Longrightarrow$ $h = w$

$2wh = lh$ $\quad l = 2w$

$2w^2 = lw$

$2w = l$ $\quad 2w + 2w + 2w = 108$

$\boxed{W = 18 \quad h = 18 \quad l = 36}$