

Name: SOLUTIONS

Quiz #3 - January 28, 2004

1. Consider the differential equation:

$$\frac{dy}{dx} = \frac{2x+1}{1+y^3}, \quad y(1) = 0$$

a. Solve the differential equation. (You don't need to solve for  $y(x)$  explicitly.)

$$(1+y^3)dy = (2x+1)dx$$

$$y + \frac{1}{4}y^4 = x^2 + x + C$$

$$y=0 \quad x=1$$

$$0 = 2 + C$$

$$C = -2$$

$$\boxed{y + \frac{1}{4}y^4 = x^2 + x - 2}$$

b. Determine the interval of validity for the solution by finding any vertical tangent line.

vert. tangent at  $y = -1$ , need  $x$ .

$$-1 + \frac{1}{4} = x^2 + x - 2$$

$$x^2 + x - \frac{5}{4} = 0$$

$$x = \frac{-1 \pm \sqrt{6}}{2}$$

both solutions are  $< 1$

$$\text{so } \boxed{x \in \left( \frac{-1 + \sqrt{6}}{2}, \infty \right)}$$

2. Solve the given differential equation:

$$y' + y^2 \sin x = 0$$

$$\frac{dy}{dx} + y^2 \sin x = 0$$

$$\frac{1}{y^2} dy + \sin x dx = C$$

$$\boxed{-\frac{1}{y} - \cos x = C}$$