Name: SOLUTIONS

Quiz #9 April 8, 2005

1. Find the inverse Laplace transform of:

$$F(s) = e^{-s}\frac{s-3}{s^2-6s+5}.$$

$$\frac{s-3}{s^2-6s+5} = \frac{s-3}{(s-3)^2-4} \quad \text{but } \mathcal{L}^{-1}\left\{\frac{u}{u^2-4}\right\} = \cosh(2u)$$

so by line 14 $\mathcal{L}^{-1}\left\{\frac{(s-3)}{(s-3)^2-4}\right\} = e^{3t}\cosh 2t$

so by line 13: $\boxed{u_1(t)e^{3(t-1)}\cosh(2(t-1))}$

2. Solve:

$$y'' + y = f(t),\ \ y(0) = 1,\ \ y'(0) = 1,\ \ f(t) = \begin{cases} 1, & 0 \le t < \pi/2 \\ 0, & t \ge \pi/2 \end{cases}$$

Notice $f(t) = 1 - u_{\pi/2}(t)$ so $\mathcal{L}\{f\} = \frac{1}{s} - \frac{1}{s}e^{-\pi/2\, s}$

$$s^2\mathcal{L}(y) - s - 1 + \mathcal{L}(y) = \frac{1}{s} - \frac{1}{s}e^{-\pi/2\, s}$$

$$\mathcal{L}(y) = \frac{s+1}{s^2+1} + \frac{1}{s(s^2+1)} - \frac{1}{s(s^2+1)}e^{-\pi/2\, s}$$

by part fractions: $\mathcal{L}(y) = \frac{s}{s^2+1} + \frac{1}{s^2+1} + \frac{1}{s} - \frac{s}{s^2+1} - \left(\frac{1}{s} - \frac{s}{s^2+1}\right)e^{-\pi/2\, s}$

$$y = \cos t + \sin t + 1 - \cos t - u_{\pi/2}(t)(1 - \cos(t - \tfrac{\pi}{2}))$$

$$y = 1 + \sin t - u_{\pi/2}(t)(1 - \cos(t - \tfrac{\pi}{2}))$$