

**Homework # 11- Due Tuesday 2/28/06, Assigned Tuesdays  
2/21/06**

0. Read p.78-87, Definitions: group homomorphism, 1-1, onto, image, preimage, kernel.

1. Exercise 4.8

2. Let  $\phi : G \rightarrow H$  be a group homomorphism. Prove that  $\phi[G]$  is abelian if and only if for all  $x, y \in G$  we have  $xyx^{-1}y^{-1} \in \text{Ker}(\phi)$ .

3. Let  $\phi : G \rightarrow H$  be a group homomorphism. Prove that  $\phi$  is 1 - 1 if and only if  $\text{Ker}(\phi) = \{e\}$ . Hint: If  $\phi(x) = \phi(y)$  then you can easily show that  $xy^{-1} \in \text{Ker}(\phi)$ .

4. Let  $G$  be a group and fix a  $g \in G$ . Define a map  $\phi : G \rightarrow G$  by:

$$\phi(x) = gxg^{-1}.$$

a. Show that  $\phi$  is 1 - 1 and onto.

b. Prove that  $\phi$  is a homomorphism.