

**Homework # 13- Due Tuesday 3/14/06, Assigned Tuesday
2/28/06**

0. Read Chapter 5. Vocabulary: simple group
1. Prove that a finite simple group G which is abelian must be a cyclic group of prime order. Hint: Pick $g \neq e$ and consider $\langle g \rangle \leq G$.
2. Suppose $H \leq G$ and $N \trianglelefteq G$. Define a subset $HN \subset G$ by:

$$HN = \{hn \mid h \in H, n \in N\}.$$

Prove that HN is a subgroup of G .

3. Suppose in Problem 2 you also assume $H \trianglelefteq G$. Now show that $HN \trianglelefteq G$.
4. Let G be a group. Define a relation \sim (called conjugacy) on G by:

$$x \sim y \text{ if there exists } g \in G \text{ such that } gxg^{-1} = y.$$

- a. Prove this is an equivalence relation. The equivalence classes are called the *conjugacy classes* of G .
- b. Describe the conjugacy classes of G if G is abelian.
- c. Write down the conjugacy classes of S_3 .
- d. Bonus: Let $H \leq G$. Prove H is normal if and only if H is a union of conjugacy classes of G .