Homework # 19- Due Tuesday 4/18/06, Assigned 4/11/06

0. Read Chapter 8. Vocabulary: finite extension, degree [E:F], splitting field, characteristic 0

1. Book 7-15, 7-16

2. Is $\mathbb{Q}(\pi) : \mathbb{Q}$ a finite extension? If so give a basis of $\mathbb{Q}(\pi) : \mathbb{Q}$. If not, clearly explain why not.

3. For each $\alpha \in \mathbb{C}$ find the minimal polynomial of α in $\mathbb{Q}[x]$ and the degree of $\mathbb{Q}(\alpha) : \mathbb{Q}$. Be sure in each case you would be able to prove your polynomial is irreducible if asked.

a. $\alpha = \sqrt{3 - \sqrt{6}}$
b. $\alpha = \sqrt{\frac{1}{3} + \sqrt{7}}$
c. $\alpha = \sqrt{2} + i$
d. $\alpha = \sqrt{2} + \sqrt{5}$

4. For each $\alpha \in \mathbb{C}$ and each given field F, classify α as either algebraic or transcendental over F. if α is algebraic, find the degree of $F(\alpha) : F$.

a. $\alpha = i, F = \mathbb{Q}$
b. $\alpha = \pi$, $F = \mathbb{Q}$
c. $\alpha = \pi^2$,$F = \mathbb{Q}(\pi)$.
d. $\alpha = \pi^2$, $F = \mathbb{Q}(\pi^3)$

5. Find the degree and a basis for the given field extension:

a. $\mathbb{Q}(\sqrt{2}) : \mathbb{Q}$
b. $\mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{18}) : \mathbb{Q}$.
c. $\mathbb{Q}(\sqrt[3]{2}, \sqrt{3}) : \mathbb{Q}$.
d. $\mathbb{Q}(\sqrt{2}, \sqrt{3}) : \mathbb{Q}(\sqrt{2} + \sqrt{3})$.
e. $\mathbb{Q}(\sqrt{2} + \sqrt{3}) : \mathbb{Q}(\sqrt{3})$.

6. Find all conjugates for $\alpha = \sqrt{2} + \sqrt{3}$ over $\mathbb{Q}$. Repeat for $\alpha = \sqrt[3]{2}$.