

Homework # 20- Due Tuesday 4/18/06, Assigned 4/13/06

0. Vocabulary: field automorphism, Galois Group, normal extension, fixed field.

1. Let $\sigma, \tau : F \rightarrow F$ be two field automorphisms. Prove that σ^{-1} and $\sigma \circ \tau$ are also field automorphisms.

2. Prove that $G(\mathbb{C}/\mathbb{R})$ is cyclic with 2 elements.

3. Let F be a field. Define the *characteristic* of F to be the smallest $n \geq 1$ such that:

$$1 + 1 + \cdots + 1 = 0$$

where we are adding 1 to itself n times. If no such n exists, we say F has *characteristic zero*.

- What is $\text{char}(\mathbb{Q})$?
- What is $\text{char} \mathbb{Z}_7$?
- Suppose F has finite characteristic n . Prove that n must be prime. Hint: Fields have no zero divisors.