Homework # 4- Due Tuesday 1/24/06, Assigned Thursday 1/19/06

0. Read p. 59-65

1. Exercise 3-15.

2. Let G be a finite group and $g \in G$. Prove that $g^{-1} = g^a$ for some $a \geq 0$, i.e. the inverse of g is some positive power of g. (Hint: Use something you proved on HW#3). Give an example to demonstrate this is not always true when G is infinite.

3. Let $g \in G$. The *order* of g is defined to be the *smallest* positive integer n such that $g^n = e$, if such an n exists. If not we say g has infinite order. For example the permutation $\sigma = (1234)$ has order 4 since $\sigma^4 = e$ but σ, σ^2, and σ^3 are not the identity. Determine the order of each of the following permutations:

$$(12345)$$
$$(123)(45)$$
$$(123)(456)(78)$$
$$(12)(34)(56)$$
$$(123)(456789)$$
$$(1234567)(8,9,10)(11,12,13,14,15,16)(17,18)$$

4. Let V consist of the following four elements of S_4:

$$V = \{e, (12)(34), (13)(24), (14)(23)\}$$

Check that V is a subgroup of S_4 and write down the multiplication table.

5. Repeat Ex. 4 for

$$W = \{e, (12), (34), (12)(34)\}$$

and

$$U = \{e, (1234), (13)(24), (1432)\}$$

6. Write $\sigma = (1234)(567)(89)$ as a product of transpositions. Is σ even or odd? Repeat for $\tau = (1234567)$.

7. Do the following multiplications. Your answer should be in disjoint cycle notation:

$$(12)(234)(34)(15)$$
$$(1234)(14)(23)(1432)$$
$$(15)(14)(13)(12)$$