Homework # 6- Due Tuesday 2/7/06, Assigned Tuesday 1/31/06

0. Vocabulary: normal subgroup, quotient group, index

1. Exercise 3-50. Hint: Notice from the table we can already determine that $a^4 = e$ and since $b^2 = a^2$ then $b^4 = e$. Thus $a^{-1} = a^3$ and we see that $ba = a^{-1}b$. This is enough information to complete the entire multiplication table.

2. Show that the quaternion group Q is not isomorphic to the dihedral group D_4.

3. 3-52, 3-53

4. We already know S_n can be generated by transpositions, i.e. every $\sigma \in S_n$ can be written as a product of transpositions. It is actually the case that S_n can be generated by *adjacent transpositions*, i.e. those of the form $(i, i+1)$. Prove this for $n = 5$ by expressing each $(a, b) \in S_5$ as a product of adjacent transpositions.

5. Prove that the intersection of two subgroups of G is also a subgroup. Give an example to demonstrate that the union of two subgroups need not be a subgroup.

6. Exercise 3-45.

7. In this exercise we will show that the converse to Lagrange's Theorem is false. In particular that A_4 is a group with 12 elements that contains no subgroup with 6 elements.

a. Write down the twelve permutations in A_4.

b. Show that any subgroup with 6 elements must contain a 3-cycle.

c. Using # 6 show that any normal subgroup containing a 3-cycle must contain all 8 3-cycles, and thus must be all of A_4.