HW #9 Solutions

1. Let G be abelian, $H = \{g \in G \mid g^2 = e\}$

Let $g_1, g_2 \in H$. Then $(g_1 g_2)^2 = g_1 g_2 g_1 g_2$

$= g_1^2 g_2^2$ since G is abelian

$= e$. Thus $g_1 g_2 \in H$.

Also $g_1 g_1 = e \Rightarrow g_1^{-1} g_1^{-1} = e \Rightarrow g_1^{-1} \in H$

Thus $H \leq G$. //

Suppose $G = S_3$ then $H = \{e, (12), (13), (23)\}$

is not a subgroup.

2. Let $x, y \in H_a$ so $xa = ax$ and $ya = yx$.

Then $(xy)a = xay = axy$ so $xy \in H_a$.

Also $xa = ax \Rightarrow x^{-1}xa = x^{-1}ax$

$a = x^{-1}ax$

$ax^{-1} = x^{-1}a$ Thus $x^{-1} \in H_a$,

so $H_a \leq G$.

3. $(12)(23) \neq (23)(12)$ so S_n is nonabelian for $n \geq 3$

4. $G = \mathbb{Z}$ has no elements of finite order > 1.

$H = 3\mathbb{Z}$

G/H has 3 elements so all elements have finite order.

5.

a. Let $x, y \in gHg^{-1}$, so $x = gh_1g^{-1}$, $y = gh_2g^{-1}$ for some $h_1, h_2 \in H$.

$xy = gh_1g^{-1}gh_2g^{-1} = g(h_1h_2)g^{-1} \in gHg^{-1}$ since $h_1h_2 \in H$.

$x^{-1} = gh_1^{-1}g^{-1} \in gHg^{-1}$ since $h_1^{-1} \in H$.

Thus $gHg^{-1} \leq G$.

b. Let $H = \{h_1, \ldots, h_n\}$. Then $gHg^{-1} = \{gh_1g^{-1}, \ldots, gh_ng^{-1}\}$.

If $gh_ig^{-1} = gh_jg^{-1}$ then $h_i = h_j$ by cancellation rules.

Thus $|gHg^{-1}| = n = |H|$.

c. gHg^{-1} is a subgroup w/ $|H|$ elts. But we are assuming H is the unique such subgroup.

Thus $gHg^{-1} = H$ $\forall g \in G$

so $H \trianglelefteq G$.

<u>HW #10</u>

comm. ring w/ identity $\mathbb{Z}$

comm ring w/out identity $2\mathbb{Z}$

noncomm ring w/ identity $M_3(\mathbb{R})$

noncomm ring w/out identity $M_3(2\mathbb{Z})$

int. domain not a field $\mathbb{Z}$

finite field $\mathbb{Z}/3\mathbb{Z}$