Math 6980 Homework # 3, Assigned 1/17/06, Due 1/24/06

1. Prove that a $\mathbb{C}G$ module U is irreducible if and only if U^* is.

2. Let G be a finite group and let $Z(\mathbb{C}(G))$ be the center of the group algebra:

$$Z(\mathbb{C}(G)) = \{z \in \mathbb{C}(G) \mid zx = xz \forall x \in \mathbb{C}(G)\}.$$

Let V be an irreducible $\mathbb{C}G$-module and let $z \in Z(\mathbb{C}(G))$. Prove that there exists a scalar $\lambda \in \mathbb{C}$ such that $zv = \lambda v \forall v \in V$. (Remark: The group algebra has a nontrivial center even if the group itself does not!)

3. Let $G = D_3 = \{r, s \mid r^3 = s^2 = e, srs = r^{-1}$. Let $\omega = e^{2\pi i/3}$ and consider the two-dimensional subspace of the regular $\mathbb{C}G$ module:

$$W = \text{span} < e + \omega^2 r + \omega r^2, s + \omega^2 rs + \omega r^2 s > .$$

a. Verify that W is a submodule of $\mathbb{C}(G)$ which is irreducible.

b. Check that $r + r^{-1} \in Z(\mathbb{C}(G))$.

c. Find the scalar λ that $r + r^{-1}$ acts on W by (see #2).

4. Fulton-Harris Exercises 1.10 - 1.14.