

Math 8300 HW Assigned 9/14/06, Due 9/19/06

1. p. 117 #7, 8, 10

2. Let  $G = GL_2(3)$  and  $K = Z(G)$ . We already know the order of  $GL_n(q)$ , in this case  $|G| = 48$  and  $|K| = 2$ .

Let

$$H = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \mid a, b, c \in F, ac \neq 0 \right\}.$$

a. Show that  $K \leq H \leq G$  and  $|H| = 12$ .

b. Prove that  $H_G = K$ .

c. Deduce that  $G/K \cong S_4$ .

3. Let  $G$  be finite with  $H, K \leq G$ . Prove that the action of  $G$  by left multiplication on  $G/H$  is equivalent to that on  $G/K$  if and only if there is an automorphism  $\phi$  of  $G$  such that  $\phi(H) = K$ .