Final Exam, Math 2850

Instr: Denis White Name ____________________

There are 200 possible points. A non graphing calculator and a formula sheet are allowed. Check that there are 8 (2 sided) pages.

1. Find an equation for the tangent plane to the surface

$$x \cos z + y^2 e^{xz} = 4 \quad \text{at} \quad P_0(3, -1, 0)$$

(12)

2. Solve the initial value problem for $\vec{r}$ as a vector function of t.

(15)

$$\begin{aligned} \frac{d\vec{r}}{dt} &= \frac{1}{(t+1)^2}\vec{i} + \frac{1}{t+1}\vec{j} + (t+1)^{1/2}\vec{k} \\ r(\vec{0}) &= \vec{j} + 2\vec{k} \end{aligned}$$

3. (a) Find the directional derivative of f at (0,1) in the direction of the vector $\vec{v} = \vec{i} + 2\vec{j}$.

(16)

$$f(x, y) = x^2 y + y^2 + y e^{xy}$$

   (b) Find the maximum rate of change of f at (0,1) and the direction in which it occurs if f is the function in part (a).

4. Find the local maximum, minimum and saddle points for the function $f(x, y) = 6x^2 - 2x^3 + 3y^2 + 6xy$.

(18)

5. Evaluate the integral $\iiint_D 4y \, dV$ if D is bounded by the elliptic paraboloid $z = 3x^2 + y^2 + 1$ and by the planes $z = 0$, $y = 2x$, $y = 0$ and $x = 1$.

(16)

6. Let D be the solid that lies between the spheres $x^2+y^2+z^2 = 1$ and $x^2+y^2+z^2 = 9$ in the first octant. Express $\iiint_D xz \, dV$ as an iterated (triple) integral in spherical coordinates. Do NOT evaluate.

(16)

7. (a) Show that $\vec{F}(x, y) = (ye^x + \sin y)\vec{i} + (e^x + x \cos y + 2y)\vec{j}$ is conservative and find a function f so that $\vec{F} = \nabla f$.

(18)

   (b) Find the work done by $\vec{F}$ ($\vec{F}$ as in part (a)) in moving an object along a curve C from $(0, 1)$ to $(2, \pi)$.

8. Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ if $\vec{F}(x, y, z) = xyz\vec{i} - xy\vec{j} + x^2\vec{k}$ along the path C given by $\vec{r}(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k}$, $0 \le t \le 2$.

(14)

9. Use Green's Theorem to evaluate the line integral

$$\int_C (xe^x + 4x^3 y) \, dx + (x^4 + 2xy) \, dy$$

where C is the boundary of the triangle $0 \le x \le 2y$, $0 \le y \le 1$ and is positively oriented.

(16)

10. Let $\vec{F} = (x^2 + xz)\vec{i} + (xy - y^3)\vec{j} + (xz + \sqrt{z})\vec{k}$

(27)

   (a) Find the curl of $\vec{F}$ (that is $\nabla \times \vec{F}$).

   (b) Find the work done by $\vec{F}$ around the curve C which is the boundary of the triangle cut from the plane $x + y + z = 1$ by the first octant and oriented counterclockwise when viewed from above. Use Stoke's Theorem.

11. Find the area of the cap of the sphere $x^2+y^2+z^2 = 2$ cut by the cone $z = \sqrt{x^2 + y^2}$

(16)

12. Find the flux of $F(x, y, z) = 4x\vec{i}+4y\vec{j}+2\vec{k}$ outward (away from the z-axis). through the surface cut from the bottom of the paraboloid $z = x^2 + y^2$ by the plane $z = 1$.

(16)