Review for Math 2850

Thomas's Calculus 12th ed.

1. Quadric and cylinder surfaces. Ellipsoid; hyperboloid of one sheet; or of two sheets; elliptic cone; elliptic paraboloid; hyperbolic paraboloid.

2. Parametric curves $\vec{r}(t)$ and velocity and tangent lines and acceleration.

3. Integrate $\int_a^b \vec{r}(t)\,dt$

4. Arclength. $\int_a^b |\vec{r}\,'(t)|\,dt$ (Formula sheet.)

5. Unit tangent vector $\vec{T}(t) = \vec{r}\,'(t)/|\vec{r}\,'(t)|$.

6. Level Curves and Contour Maps (p 773, no. 60)

7. Limits, Continuity, Partial Derivatives and Differentiability.

8. Higher Order and Mixed Partials

9. Differentials.

10. Chain Rule Page 801, 43.

11. Directional Derivatives and Gradient. Page 808-809, 11-24.

12. Tangent Plane to $z = f(x, y)$ or to $F(x, y, z) = C$. Page 646, 31-34.

$$z = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

$$F_x(a, b, c)(x - a) + F_y(a, b, c)(y - b) + F_z(a, b, c)(z - c) = 0$$

(Formula sheet.)

13. Local Maxima, Minima and Saddles on open or unbounded sets. Page 652, 3-14.

14. Absolute Maximum and Minimum on closed bounded sets. Check for critical points in the interior and then check the boundary. The check of the boundary requires working around corners.

15. Double integrals and volume under the graph of $z = f(x, y)$ and above the region D in the xy-plane.

16. Double integrals as iterated integrals (Fubini's theorem) and evaluation.

17. Polar Coordinates and integration in polar coordinates. $r = a\cos\theta$ or $r = a\sin\theta$ (circles); $r = a(1 \pm \cos\theta)$ or $r = a(1 \pm \sin\theta)$ (cardioids)

18. Mass and moments in 2 dimensions.

19. Mass $\iiint_D \delta\,dV$ as a triple integral.

20. Evaluation of triple integrals using Fubini's Theorem and iterated integrals.

21. Cylindrical Coordinates to evaluate triple integrals. Page 904 55

22. Spherical Coordinates to evaluate triple integrals. Page 904 35

23. Work done (or circulation) of a vector field $\vec{F} = M\vec{i} + N\vec{j} + P\vec{k}$ a force along a curve C paramterized by $\vec{r}(t)$, $a \le t \le b$. p936, 19-22.

$$\int_C \vec{F} \cdot d\vec{r} = \int_C M\,dx + N\,dy + P\,dz = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t)\,dt$$

24. Flux of a vector field $\vec{F} = M\vec{i} + N\vec{j}$ in the plane outward through a closed curve is $\int_C M\,dy - N\,dx$

25. Conservative Vector Fields $\vec{F}(x, y, z)$, that is $M_y = N_x$, $M_z = P_x$ and $N_z = P_y$. If $\vec{F}$ is conservative on a simply connected domain then there is $f(x, y, z)$ so that $\vec{F} = \nabla f$. p947, 10

26. Green's Theorem. Work done by $\vec{F} = M\vec{i} + N\vec{j}$ around a closed curve C in the xy-plane that bounds a region D and is oriented counterclockwise.

$$\int_C F \cdot d\vec{r} \left(= \int_C M\,dx + N\,dy \right) = \iint_D \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\,dA$$

27. Green's Theorem (Divergence form)

$$\int_C M\,dy - N\,dx = \iint_D \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\,dA$$

Page 958, 9.

28. Surfaces S and their parameterizations $\vec{r}(u, v)$, (u, v) belongs to some set R in the uv-plane.

29. Surface area: $\iint_R \left| \frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v} \right| dA$ Notation $\iint_{\S} d\sigma$.

30. If the surface is $z = f(x, y)$, for (x, y) in R then the surface area is

$$\iint_R \left[1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 \right]^{1/2} dA$$

31. Normal to the surface

$$\vec{N} = \frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v}$$

and the unit normal is $\vec{n} = \vec{N}/|\vec{N}|$

32. Flux through a surface.

$$\iint_S \vec{F} \cdot \vec{n}\, d\sigma = \iint_D \vec{F} \cdot \left(\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v} \right) dA$$

Page 979, 19 to 42.

33. The divergence of a vector field $\vec{F} = M\vec{i} + N\vec{j} + P\vec{k}$ is $\nabla \cdot \vec{F} = M_x + N_y + P_z$. (Page 999, 1-4)

34. The curl of a vector field $\vec{F} = M\vec{i} + N\vec{j} + P\vec{k}$ is $\nabla \times \vec{F} = (P_y - N_z)\vec{i} + (M_z - P_x)\vec{j} + (N_x - M_y)\vec{k}$. (Example: Find the curl of $\vec{F} = x^2 y\vec{i} + (2y^3 z - 5x)\vec{j} + 3z\vec{k}$.)

35. The Divergence Theorem. If D is some solid in 3-space and S is its boundary oriented outward

$$\iint_S \vec{F} \cdot \vec{n}\, dS = \iiint_D \nabla \cdot \vec{F}\, dV$$

Page 999, Problems 5-16.