2 Pages! **Quiz 2**, Math 2850-005
9/8/2016 Solutions Name ______________________

1. The position of a particle at time t is

$$\vec{r}(t) = (2\ln(t+1))\vec{i} + t^2\vec{j} + (t^2/2)\vec{k}.$$

(5) Find the particle's velocity and acceleration at time $t = 1$.

The velocity is

$$\vec{v}(t) = \vec{r}'(t) = \frac{2}{t+1}\vec{i} + 2t\vec{j} + t\vec{k}.$$

and the acceleration is

$$\vec{a}(t) = \vec{v}'(t) = -\frac{2}{(t+1)^2}\vec{i} + 2\vec{j} + \vec{k}.$$

At time $t = 1$ we have the velocity is $\vec{v}(1) = \vec{i} + 2\vec{j} + \vec{k}$ and the acceleration is $\vec{a}(1) = (1/2)\vec{i} + 2\vec{j} + \vec{k}$.

2. Find parametric equations for the line that is tangent to the curve

$$\vec{r}(t) = (\sin t)\,\vec{i} + (t^2 - \cos t)\vec{j} + e^t\,\vec{k}$$

(5) at the parameter value $t = 0$.

The vector $\vec{r}'(t)$ when $t = 0$ is a direction vector for the tangent to the curve.

$$\vec{v}(t) = (\cos t)\vec{i} + (2t + \sin t)\vec{j} + e^t\vec{k}$$

so that $\vec{v}(0) = \vec{i} + \vec{k}$. Also $\vec{r}(0) = -\vec{j} + \vec{k}$ and so the parametric equations for the tangent line are

$$\vec{R}(t) = -\vec{j} + \vec{k} + t(\vec{i} + \vec{k}) = t\vec{i} - \vec{j} + (1+t)\vec{k}$$

3. Solve the initial value problem for $\vec{r}$ as a vector function of t.

$$\frac{d\vec{r}}{dt} = \frac{3}{2}(t+1)^{1/2}\vec{i} + e^{-t}\vec{j} + \frac{1}{t+1}\vec{k}$$
$$\vec{r}(0) = \vec{k}$$

(5)

By the fundamental theorem of calculus (in vector form)

$$\vec{r}(t) - \vec{r}(0) = \int_0^t \frac{d\vec{r}}{dt}(\tau)\,d\tau$$

Therefore

$$\vec{r}(t) - \vec{k} = \int_0^t \frac{3}{2}(t+1)^{1/2}\vec{i} + e^{-t}\vec{j} + \frac{1}{t+1}\vec{k}\,dt$$
$$= (\tau+1)^{3/2}\vec{i} - e^{-\tau}\vec{j} + \ln(\tau+1)\vec{k}|_0^t$$
$$= ((t+1)^{3/1} - 1)\vec{i} - (e^{-t} - 1)\vec{j} + \ln(t+1)\vec{k}$$

so that $\vec{r}(t) = ((t+1)^{3/2} - 1)\vec{i} + (1 - e^{-t})\vec{j} + (\ln(t+1) + 1)\vec{k}$.

4. Find the length of the curve.

$$\vec{r}(t) = (\cos^3 t)\vec{i} + (\sin^3 t)\vec{j}, \quad 0 \le t \le \pi/2$$

(5)

The length of the curve is

$$\int_0^{\pi/2} |\vec{r}'(t)|\,dt = \int_0^{\pi/2} = [(-3\cos^2 t \sin t)^2 + (3(\sin t)^2 \cos t)^2]^{1/2}\,dt \quad (1)$$
$$= \int_0^{\pi/2} [9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t)]^{1/2}\,dt \quad (2)$$
$$= \int_0^{\pi/2} [9\cos^2 t \sin^2 t(\cos^2 t + \sin^2 t)]^{1/2}\,dt \quad (3)$$
$$= \int_0^{\pi/2} 3\cos t \sin t\,dt \quad (4)$$
$$= \frac{3}{2}(\sin t)^2|_0^{\pi/2} = \frac{3}{2} \quad (5)$$

where the final integration can be done, for example by substitution $u = \sin t$, $du = \cos t\,dt$