2 Pages!
9/15/2016

Quiz 3, Math 2850-005
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1. Find the unit tangent vector $\vec{T}(t)$ for the space curve $\vec{r}(t) = (e^t \cos t)\vec{i} + (e^t \sin t)\vec{j} + 2\vec{k}$

(5)

Recall that the unit tangent vector is $\vec{T}(t) = \vec{r}'(t)/|\vec{r}'(t)|$. Since $\vec{r}'(t) = e^t(\cos t - \sin t)\vec{i} + e^t(\sin t + \cos t)\vec{j}$ so that

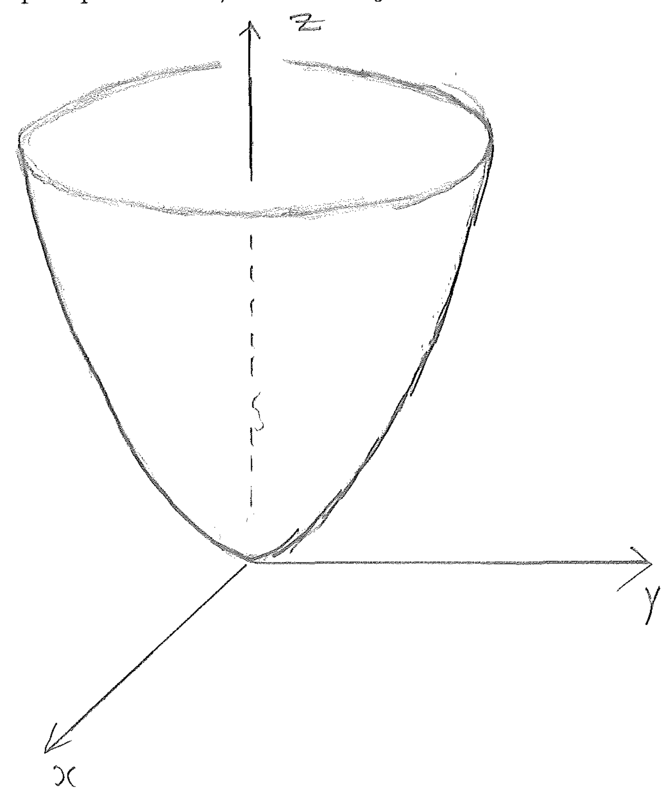
$$|\vec{r}'(t)| = \left(e^{2t}(\cos t - \sin t)^2 + e^{2t}(\sin t + \cos t)^2\right)^{1/2}$$
$$= e^t\left(\cos^2 t + \sin^2 t - 2\cos t \sin t + \sin^2 t + \cos^2 t + 2\sin t \cos t\right)^{1/2} = \sqrt{2}e^t$$

and so
$$\vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|} = \frac{1}{\sqrt{2}}\left((\cos t - \sin t)\vec{i} + (\sin t + \cos t)\vec{j}\right)$$

2. Display the values of $f(x, y) = 4x^2 + y^2$ in two ways:

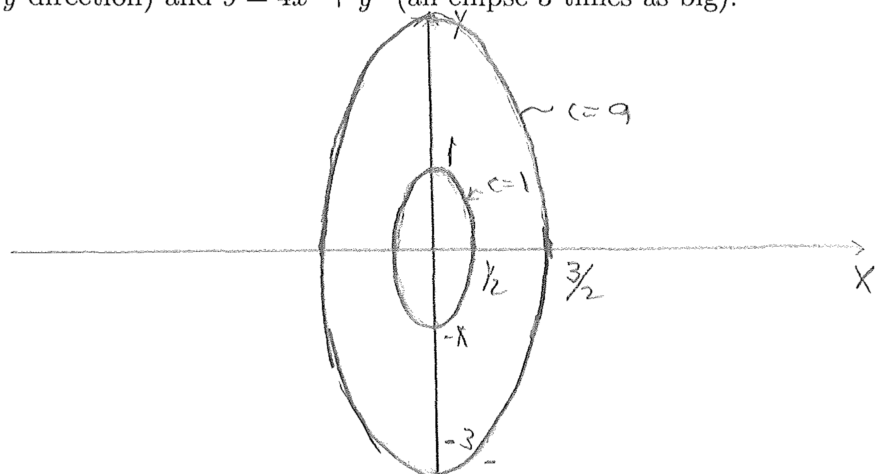
(5)

   (a) by sketching the surface $z = f(x, y)$ and
   The graph is an elliptic paraboloid, $z = 4x^2 + y^2$



(b) by drawing an assortment of level curves (at least 2) in the functions domain. Label each curve with its function value.

(5) We graph the two level curves $1 = 4x^2 + y^2$ (an ellipse with major axis one in the y direction) and $9 = 4x^2 + y^2$ (an ellipse 3 times as big).



3. Sketch a typical level **surface** of $f(x, y, z) = x^2 + y^2$

(5) A typical level surface is $f(x, y, z) = c$ or $x^2 + y^2 = c$ for $c > 0$ and that is a circular cylinder centered on the z–axis and radius $\sqrt{c}$. When $c = 1$ we get the surface below.