2 Pages! **Quiz 4**, Math 2850-005

9/22/2016 Solutions Name ____________________

(4) 1. Find the limit by rewriting the fraction: $\displaystyle\lim_{(x,y)\to(2,0)} \frac{\sqrt{2x-y}-2}{2x-y-4}$

Factor the divisor as $2x-y-4 = (\sqrt{2x-y}-2)(\sqrt{2x-y}+2)$:

$$\begin{aligned}\lim_{(x,y)\to(2,0)} \frac{\sqrt{2x-y}-2}{2x-y-4} &= \lim_{(x,y)\to(2,0)} \frac{\sqrt{2x-y}-2}{(\sqrt{2x-y}-2)(\sqrt{2x-y}+2)}\\ &= \lim_{(x,y)\to(2,0)} \frac{1}{\sqrt{2x-y}+2} = \frac{1}{\sqrt{4-0}+2} = \frac{1}{4}\end{aligned}$$

For the given expression to exist it is assumed that $2x-y>0$ (so that the square root exists) and $2x-y\neq 4$ (so that the fraction exists).

2. By considering different paths of approach show that the function $g(x,y) = \dfrac{x-y}{x+y}$

(4) has no limit as $(x,y)\to(0,0)$.

Straight lines through the origin should work: Try $y=0$ where $\dfrac{x-y}{x+y}=1$ and $x=0$ where $\dfrac{x-y}{x+y}=-1$. Therefore the limiting value of $\dfrac{x-y}{x+y}$ is 1 along $y=0$ (as $x\to 0$) and is -1 along $x=0$ (as $y\to 0$) and these limiting values are not the same and so the overall limit

$$\lim_{(x,y)\to(0,0)} \frac{x-y}{x+y} \quad \text{does not exist}$$

(3) 3. At which points in the plane is the function $f(x,y) = \dfrac{x+y}{x-y}$ continuous?

The function is only defined when $y\neq x$ and so it can only be continuous at $y\neq x$ because there is no division by 0 and all operations (like addition and subtraction are continuous.

(3) 4. Find $\partial f/\partial x$ if $f(x,y) = \dfrac{x+y}{xy-1}$

By the quotient rule

$$\frac{\partial f}{\partial x} = \frac{xy-1-y(x+y)}{(xy-1)^2} = \frac{-1-y^2}{(xy-1)^2}$$

(6) 5. Find all the second-order partial derivatives of $f(x,y) = x\sin(x^2y)$.

We need the first partials:

$$\begin{aligned}\frac{\partial f}{\partial x} =& \sin(x^2y) + 2x^2y\cos(x^2y)\\ \frac{\partial f}{\partial y} =& x^3\cos(x^2y)\end{aligned}$$

Now we can calculate the second partials:

$$\frac{\partial^2 f}{\partial x^2} = 2xy\cos(x^2y) + 4xy\cos(x^2y) - 4x^3y^2\sin(x^2y) = 6xy\cos(x^2y) - 4x^3y^2\sin(x^2y)$$
$$\frac{\partial^2 f}{\partial x\partial y} = x^2\cos(x^2y) + 2x^2\cos(x^2y) - 2x^4y\sin(x^2y) = 3x^2\cos(x^2y) - 2x^4y\sin(x^2y)$$
$$\frac{\partial^2 f}{\partial y^2} = -x^5\sin(x^2y)$$

and of course:

$$\frac{\partial^2 f}{\partial x\partial y} = \frac{\partial^2 f}{\partial y\partial x}$$