2 Pages! **Quiz 5**, Math 2850-005
10/13/2016 Solutions Name ____________________

(6) 1. Find equations for the

(a) tangent plane and

(b) normal line at the point $P_0(3,5,-4)$ on the surface.

$$x^2+y^2-z^2=18$$

The surface (a hyperboloid of one sheet) is a level surface of $f(x,y,z)=x^2+y^2-z^2$ and so $\nabla f = 2x\vec{i}+2y\vec{j}-2z\vec{k}$ is normal to the surface. Therefore $\nabla f(3,5,-4) = 6\vec{i}+10\vec{j}+8\vec{k}$ is normal at P_0 and so the tangent planes is

$$6(x-3)+10(y-5)+8(z+4)=0 \quad \text{or} \quad 3x+5y+4z=18$$

and the normal line is

$$\vec{r}(t) = (3+6t)\vec{i}+(5+10t)\vec{j}+(-4+8t)\vec{k}$$

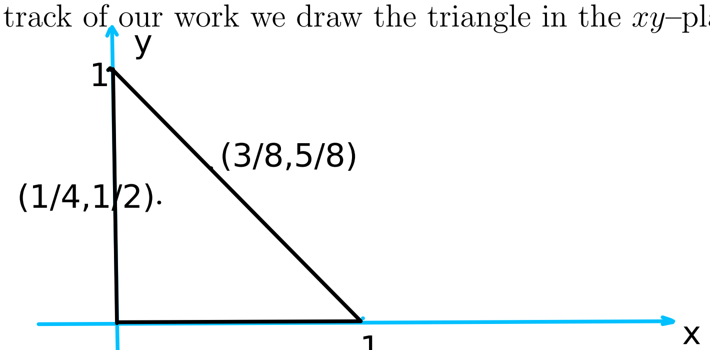
(4) 2. Find the linearization $L(x,y,z)$ of $f(x,y,z)=e^x+\cos(y+z)$ at $(0,\frac{\pi}{2},0)$.

Compute the gradient. $\nabla f = e^x\vec{i} - \sin(y+z)\vec{j} - \sin(y+z)\vec{k}$ at $(0,\frac{\pi}{2},0)$ is $\nabla f(0,\pi/2,0) = \vec{i}-\vec{j}-\vec{k}$ and $f(0,\pi/2,0)=1$ so that

$$\begin{aligned} L(x,y,z) =& f(0,\tfrac{\pi}{2},0)+f_x(0,\tfrac{\pi}{2},0)x+f_y(0,\tfrac{\pi}{2},0)(y-\tfrac{\pi}{2})+f_z(0,\pi/2,0)z \\ =& 1+x-(y-\tfrac{\pi}{2})-z \end{aligned}$$

(10) 3. Find the absolute maximum and minimum of $f(x,y)=4x-8xy+2y+1$ on the triangular plate bounded by the lines $x=0$, $y=0$ and $x+y=1$ in the first quadrant.

To keep track of our work we draw the triangle in the xy–plane.



Step I Interior: We need $\nabla f = (4-8y)\vec{i}+(2-8x)\vec{j}$. The critical points are where $\nabla f = \vec{0}$ (or where ∇f is not defined) and so $4-8y=0$ and $2-8x=0$ so that $y=1/2$ and $x=1/4$: there is one critical point (1/4,1/2) and it is in the triangle.

Step II Edges: There are three edges

i. $x = 0$, $0 \leq y \leq 1$. We want to find the maxima and minima of $f(0, y) = 2y + 1$ but this is an increasing (linear) function and so has no interior critical points and the maxima and minima occur at the endpoints $y = 0$ and $y = 1$ but these are vertices of the triangle which are dealt with in Step III.

ii. $y = 0$, $0 \leq x \leq 1$. We want to find the maxima and minima of $f(x, 0) = 4x + 1$ but again this is an increasing (linear) function and so its maxima and minima at the end points $x = 0$ and $x = 1$ which are dealt with in Step III.

iii. $x + y = 1$, $0 \leq x \leq 1$. We want to find the maxima and minima of $f(x, 1-x) = 4x - 8x(1-x) + 2(1-x) + 1 = 3 - 6x + 8x^2$, $0 \leq x \leq 1$. We differentiate to look for critical points. (This is a closed interval method of Section 4.1 from Calc I.) $(d/dx)f(x, 1-x) = -6 + 16x$ and the only critical point is at $x = 3/8$ (for set the derivative to 0). We get the point (3/8,5/8)

Step III Vertices: (0,0), (1,0), (0,1).

Step IV Evaluate at all the points discovered in Steps I through III

Point P	f(P)
$(1/4, 1/2)$	f(1/4,1/2) =2
$(3/8, 5/8)$	f(3/8,5/8)=15/8
$(0, 0)$	f(0,0)=1
$(1, 0)$	f(1,0)=5
$(0, 1)$	f(0,1)=3

Therefore the absolute maximum value of f on the triangle is 5 and it occurs at (1,0); the absolute minimum value is 1 and it occurs at (0,0). (To visualize f it is helpful to note that the critical point (1/4,1/2) is a saddle point (by the second derivative test) and so the absolute max and min both must occur elsewhere and since there are no other critical points they must occur on the boundary (edges plus vertices).)