2 Pages! **Quiz 6**, Math 2850-007
10/22/2015 Solutions Name ____________________

1. Find the local maxima and minima and saddle points of the function $f(x, y) =$
(9) $6x^2 - 2x^3 + 3y^2 + 6xy$.

We need $\nabla f(x, y) = (12x - 6x^2 + 6y)\vec{i} + (6y + 6x)$. Since f is differentiable everywhere the only critical points come when $\nabla f = \vec{0}$ so that

$$12x - 6x^2 + 6y = 0$$
$$6y + 6x = 0$$

From the second equation we have $y = -x$ and using this in the first equation gives $6x - 6x^2 = 0$ or $x(1 - x) = 0$. The critical points are (1,-1) and (0,0). At each of these we apply the second derivative test and so we need the second derivatives $f_{xx} = 12 - 12x$, $f_{xy} = 6$ and $f_{yy} = 6$. Now check each critical point

(0,0) Here $f_{xx} = 12$ and $D = f_{xx}f_{yy} - f_{xy}^2 = 36 > 0$ and $f_{xx} > 0$ so that (0,0) is a local minimum and the local minimum values is $f(0, 0) = 0$

(1,-1) Here $f_{xx} = 0$ and $D = -36 < 0$ and so (1,-1) is a saddle point. ($f(1, -1) =$ 1.)

2. Evaluate the double integral over R: $0 \le x \le 1$, $0 \le y \le 2$.

$$\iint_R \frac{xy^3}{x^2 + 1}\, dA$$

(6)

We are asked to evaluate the interated integral

$$\int_0^1 \int_0^2 \frac{xy^3}{x^2+1}\, dy\, dx = \int_0^1 \frac{1}{4} y^4|_0^2 \frac{x}{x^2+1}\, dx$$
$$= \frac{1}{4}2^4 \int_0^1 \frac{x}{x^2+1}\, dx$$

To do the integral over x we substitute $u = x^2 + 1$ so that $du = 2x\, dx$. We can convert the limits of integration if we note that $x = 0$ implies $u = 1$ and $x = 1$ implies $u = 2$

$$\int_0^1 \int_0^2 \frac{xy^3}{x^2+1}\, dy\, dx = \frac{1}{4}2^4 \int_0^1 \frac{x}{x^2+1}\, dx$$
$$= 2\int_1^2 \frac{1}{u}\, du$$
$$= 2 \ln u|_1^2 = 2 \ln 2$$

(The other order of integration is also possible and gives the same answer.)

3. Find the volume of the region bounded above by the surface $z = 2\sin x \cos y$ and
(5) below by the rectangle R: $0 \le x \le \pi/2$, $0 \le y \le \pi/4$.

Volume is given by the double integral

$$\begin{aligned}\iint_R 2\sin x \cos y \, dA &= \int_0^{\pi/2}\int_0^{\pi/4} 2\sin x \cos y \, dy \, dx \\ &= \int_0^{\pi/2} 2\sin x \sin y|_0^{\pi/4} \, dx \\ &= \sqrt{2}\int_0^{\pi/2} \sin x \, dx \\ &= \sqrt{2}\,[-\cos x]_0^{\pi/2} \\ &= \sqrt{2}\,[-\cos(\pi/2) - (-\cos 0)] = \sqrt{2}\end{aligned}$$