2 Pages! **Quiz 7**, Math 2850-005

10/27/2016 Solutions Name ____________________

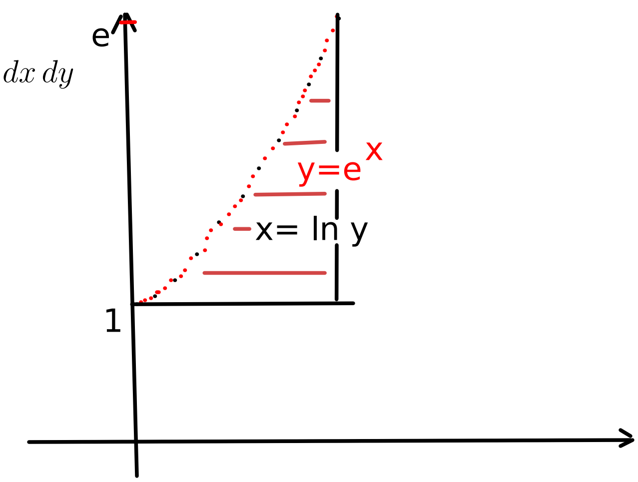
1. Sketch the region of integration and write and equivalent double integral with the order of integration reversed. You need NOT evaluate.

$$\int_0^1 \int_1^{e^x} dy\, dx$$

(4)

The region is bounded above by the curve $y = e^x$ and below by the line $y = 1$, for $0 \le x \le 1$. Reverse the order

$$\int_0^1 \int_1^{e^x} dy\, dx = \int_1^e \int_{\ln y}^1 dx\, dy$$

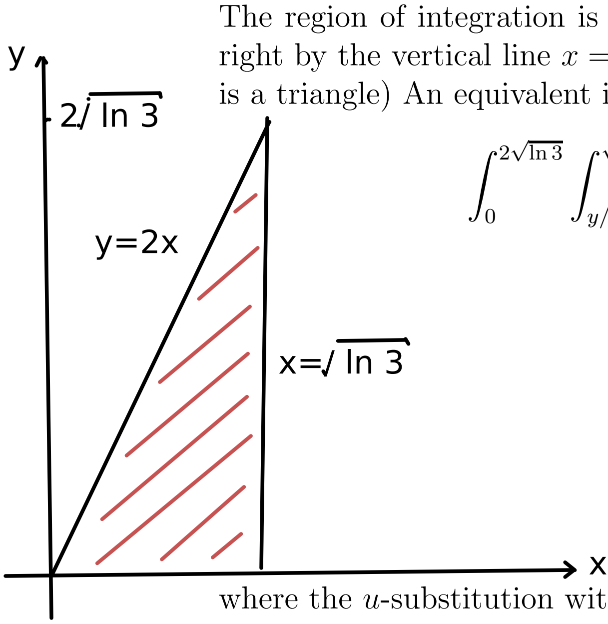


2. Sketch the region of integration, reverse the order of integration and evaluate the integral

$$\int_0^{2\sqrt{\ln 3}} \int_{y/2}^{\sqrt{\ln 3}} e^{x^2}\, dx\, dy$$

(7)

The region of integration is bounded on the left by the line $x = y/2$ and on the right by the vertical line $x = \sqrt{ln 3}$ for and $0 \le y \le 2\sqrt{\ln 3}$. Sketch the region. (It is a triangle) An equivalent integral is



$$\int_0^{2\sqrt{\ln 3}} \int_{y/2}^{\sqrt{\ln 3}} e^{x^2}\, dx\, dy = \int_0^{\sqrt{\ln 3}} \int_0^{2x} e^{x^2}\, dy\, dx$$

$$= \int_0^{\sqrt{\ln 3}} e^{x^2} y|_{y=0}^{y=2x}\, dx$$

$$= \int_0^{\sqrt{\ln 3}} e^{x^2} 2x\, dx$$

$$= \int_0^{\ln 3} e^u\, du$$

$$= e^{ln 3} - e^0 = 2$$

where the u-substitution with $u = x^2$ was applied to evaluate the last integral.

3. Change the Cartesian integral into an equivalent polar integral. Then evaluate the polar integral. (Suggestion: Sketch the region of integration.)

$$\int_0^1 \int_x^{\sqrt{2-x^2}} x + 2y\, dy\, dx$$

(9)

The region of integration is bounded below by the line $y = x$ and above by the circle $y = \sqrt{2-x^2}$ (or $x^2 + y^2 = 2$), for $0 \le x \le 1$. This is the eighth of a disk of radius $\sqrt{2}$: $0 \le r \le \sqrt{2}$ and $\pi/4 \le \theta \le \pi/2$. Therefore

$$\int_0^1 \int_x^{\sqrt{2-x^2}} x + 2y\, dy\, dx = \int_{\pi/4}^{\pi/2} \int_0^{\sqrt{2}} (r\cos\theta + 2r\sin\theta) r\, dr\, d\theta$$

Evaluate

$$\begin{aligned}\int_{\pi/4}^{\pi/2} \int_0^{\sqrt{2}} (r\cos\theta + 2r\sin\theta) r\, dr\, d\theta &= \int_{\pi/4}^{\pi/2} \frac{1}{3} r^3 |_0^{\sqrt{2}} (\cos\theta + 2\sin\theta)\, d\theta \\ &= \frac{2^{3/2}}{3} \left[\sin\theta - 2\cos\theta\right]_{\pi/4}^{\pi/2} \\ &= \frac{2^{3/2}}{3} \left[1 - (\sqrt{2}/2 - 2\sqrt{2}/2)\right] \\ &= \frac{2^{3/2}}{3} \left[(2+\sqrt{2})/2\right] = \frac{2\sqrt{2}+2}{3}\end{aligned}$$