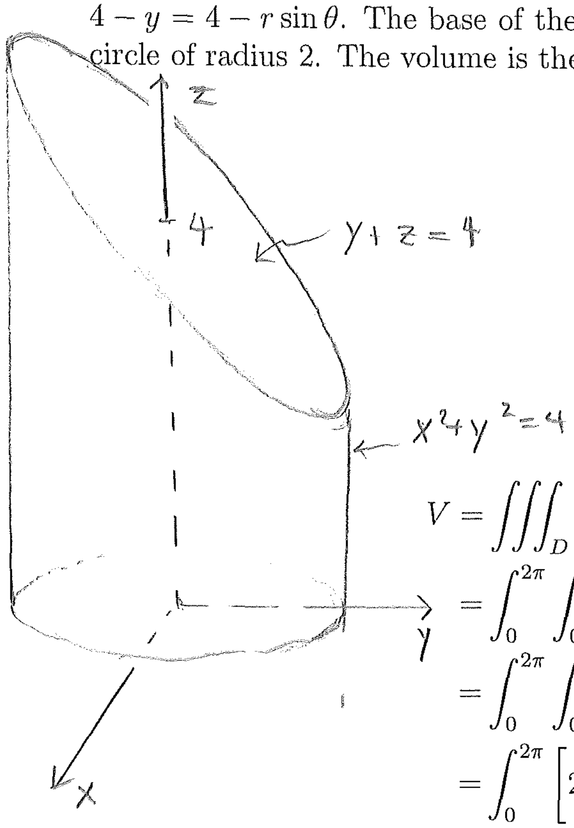
2 Pages!
11/17/2016

Quiz 8, Math 2850-005
Solutions

Name ____________________

(8) 1. Find the volume of the region enclosed by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $y + z = 4$.

This region D is best described in polar coordinates. The upper bound is $z = 4 - y = 4 - r\sin\theta$. The base of the solid in the plane $z = 0$ is the interior of the circle of radius 2. The volume is therefore



$$
\begin{aligned}
V &= \iiint_D dV \\
&= \int_0^{2\pi}\int_0^2\int_0^{4-r\sin\theta} dzr\,dr\,d\theta \\
&= \int_0^{2\pi}\int_0^2 (4 - r\sin\theta)r\,dr\,d\theta \\
&= \int_0^{2\pi}\left[2r^2 - \frac{1}{3}r^3\sin\theta\right]_0^2 d\theta \\
&= \int_0^{2\pi}\left[8 - \frac{8}{3}\sin\theta\right] d\theta \\
&= \left[8 - \frac{8}{3}\sin\theta\right]_0^{2\pi} = 16\pi
\end{aligned}
$$

(4) 2. Evaluate $\int_C x\,ds$ where C is the parabolic curve, $x = t$, $y = t^2$, from (0,0) to (2,4).

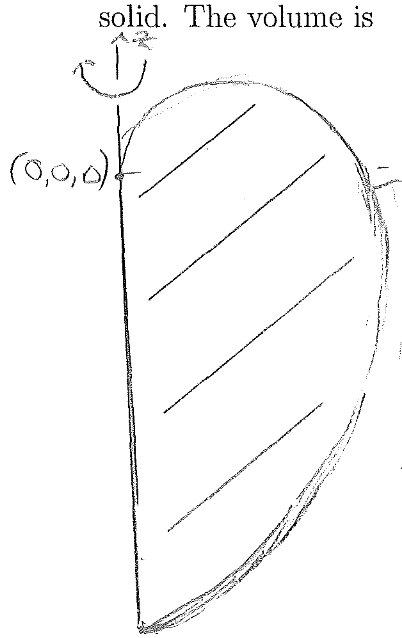
The parabolic curve is traced out over the interval $0 \le t \le 2$. We further have $ds = \sqrt{(x')^2 + (y')^2} = \sqrt{1 + (2t)^2}\,dt$. Therefore

$$\int_C x\,ds = \int_0^2 t(1+4t^2)^{1/2}\,dt = \frac{1}{8}\int_1^{17} u^{1/2}\,du = \frac{1}{8}\frac{2}{3}u^{3/2}|_1^{17} = \frac{1}{12}\left[17^{3/2} - 1\right]$$

where we have used u substitution with $u = 1 + 4t^2$ so that $(1/8)\,du = dt$.

(8) 3. Find the volume of the solid enclosed by the cardioid of revolution $\rho = 1 - \cos\phi$.

Here spherical coordinates is appropriate. Because the solid is a solid of revolution its intersection with any half plane $\theta =$ constant is illustrative of the shape of the solid. The volume is



$$
\begin{aligned}
V &= \int_0^{2\pi}\int_0^{\pi}\int_0^{1-\cos\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta \\
&= \int_0^{2\pi}\int_0^{\pi}\int_0^{1-\cos\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta \\
&= \int_0^{2\pi}\int_0^{\pi} \frac{1}{3}\rho^3 |_0^{1-\cos\phi} \sin\phi \, d\phi \, d\theta \\
&= \int_0^{2\pi}\int_0^{\pi} \frac{1}{3}(1-\cos\phi)^3 \sin\phi \, d\phi \, d\theta
\end{aligned}
$$

To evaluate the integral over ϕ we substitute $u = 1 - \cos\phi$ so that $du = \sin\phi$. After integration and substituting back we have

$$
\begin{aligned}
V &= \int_0^{2\pi} \frac{1}{12}(1-\cos\phi)^4|_0^{\pi} \, d\theta \\
&= \int_0^{2\pi} \frac{2^4}{12} \, d\theta \\
&= \frac{2^4}{12}(2\pi) = \frac{8\pi}{3}
\end{aligned}
$$