2 Pages! **Quiz 9**, Math 2850-005

12/1/2016 Solutions Name ____________________

1. Find the work done by $\vec{F} = xy\vec{i} + y\vec{j} - yz\vec{k}$ over the curve $\vec{r}(t) = t\vec{i} + t^2\vec{j} + t\vec{k}$,

(6) $0 \le t \le 1$ in the direction of increasing t.

The work is $\int_C \vec{F} \cdot d\vec{r} = \int_C M\,dx + N\,dy + P\,dz$ and $M = xy$, $N = y$ and $P = -yz$; and $x = t$, $y = t^2$ and $z = t$ so that

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^1 t(t^2)\,dt + t^2(2t)\,dt + (-t^2)(t)\,dt = \int_0^1 t^3 + 2t^3 - t^3\,dt$$
$$= \int_0^1 2t^3\,dt$$
$$= \frac{1}{2}t^4|_0^1 = \frac{1}{2}$$

2. Find the flux of $\vec{F} = x\vec{i} + y\vec{j}$ outward across the ellipse $\vec{r}(t) = \cos t\vec{i} + 4\sin t\vec{j}$,

(6) $0 \le t \le 2\pi$.

The flux is $\int_C M\,dy - N\,dx$. Along the curve $x = \cos t$ and $y = 4\sin t$ so that flux is

$$\int_C M\,dy - N\,dx = \int_0^{2\pi} \cos t(4\cos t) - 4\sin t(-\sin t)\,dt$$
$$= \int_0^{2\pi} 4\,dt = 8\pi$$

3. Evaluate the line integral by finding a potential function corresponding to the vector field

$$\int_{(1,1,1)}^{(1,2,3)} 3x^2\,dx + \frac{z^2}{y}\,dy + 2z\ln y\,dz$$

(8) (You may assume that the vector field $\vec{F} = 3x^2\vec{i} + \frac{z^2}{y}\vec{j} + 2z\ln y\vec{k}$ is conservative.)

We need the potential f so that $\nabla f = \vec{F}$. We integrate $\partial f/\partial x = 3x^2$ with respect to x regarding y and z as constants. Therefore $f(x, y, z) = x^3 + g(y, z)$ ($g(y, z)$ is the constant of integration.) Next we want $\partial f/\partial y = z^2/y$ so that $\partial g/\partial y = z^2/y$ so that $g(y, z) = z^2 \ln y + h(z)$ and $f(x, y, z) = x^3 + z^2 \ln y + h(z)$. Now we want $\partial f/\partial z = 2z\ln y$ or $2z\ln y + h'(z) = 2z\ln y$. Therefore $h'(z) = 0$ and so $h(z) = C$ is a constant. Finally we have $f(x, y, z) = x^3 + z^2 \ln y + C$. We now apply the fundamental theorem of line integrals

$$\int_{(1,1,1)}^{(1,2,3)} 3x^2\,dx + \frac{z^2}{y}\,dy + 2z\ln y\,dz = f(1, 2, 3) - f(1, 1, 1)$$
$$= 1^3 + 3^2\ln 2 + C - (1^3 + 1^2\ln 1 + C) = 9\ln 2$$