Instr: Denis White Name ______________________

(9) 1. Identify and sketch the graph of the surface $4x^2 + y^2 - z^2 = 1$.

2. The position of a particle at time t is $\vec{r}(t) = t\ln(t+1)\vec{i} + (t+1)^{1/2}\vec{j} + (t+1)^{3/2}\vec{k}$

(12) (a) Find the velocity and acceleration vectors

(b) Find an equation for the tangent line to the path (in part (a)) of the particle when $t = 0$.

(10) 3. Solve the initial problem for $\vec{r}$ as a vector function of t if

Differential equation: $\dfrac{d\vec{r}}{dt} = te^{-t}\vec{i} + e^{-t}\vec{j} - e^{-2t}\vec{k}$

Initial Condition: $\vec{r}(0) = -\vec{i} + \vec{j} + 2\vec{k}$

(8) 4. Find the length of the curve $\vec{r} = \frac{1}{3}t^{3/2}\vec{i} + 2t\vec{j} + \frac{1}{\sqrt{3}}t^{3/2}\vec{k}$, $2 \leq t \leq 7$.

5. (a) Find and sketch the domain of the function $f(x, y) = \sqrt{x^2 + 4y^2 - 9}$.

(14)

(b) Find an equation for and sketch the graph of the level curve of the function $f(x, y)$ of part (a) that passes through the point (3,-1).

(7) 6. Find the limit. $\lim_{(x,y)\to(2,-2)} \frac{x+y}{x^2-y^2}$

(9) 7. By considering different paths of approach, show that $f(x,y)$ has no limit as $(x,y) \to (0,0)$.

$$f(x,y) = \frac{y^2+xy}{x^2+y^2}$$

(9) 8. Find all the second partial derivatives of $f(x,y) = \ln(x+y^2)$

9. Suppose we substitute $x = u^2 - v^2$ and $y = 2uv$ in a differentiable function $w = f(x, y)$. Show that

$$\frac{1}{2}\frac{\partial w}{\partial u} = \frac{\partial f}{\partial x}u + \frac{\partial f}{\partial y}v$$

and derive a comparable expression for $\partial w/\partial v$.

(10)

10. (a) Find the gradient of the function $f(x, y, z) = xy + 2yz + 3xz$.

(12)

(b) Find the (directional) derivative of f at $P_0(1, 3, -1)$ in the direction of $\vec{u} = \vec{i} - \vec{j} + 2\vec{k}$.