13.4 Curvature:
The unit tangent vector is

$$\vec{T}(t) = \frac{1}{|\vec{v}(t)|}\vec{v}(t)$$

which is the direction that the partivel is traveling. $\vec{T}(t)$ is the unit vector tangent to the curve and in the direction of travel. The unit normal is $\vec{N}(t)$ defined by

$$\vec{N}(t) = \frac{T'(t)}{|T'(t)|}$$

Since $|\vec{T}| = 1$ it follows that the unit normal $N(t)$ is perpendicular to $\vec{T}(t)$ and it is not hards to see that it is in the plane determined by the velocity and acceleration. An orthogonal unit vector $\vec{B}(t)$ which is perpendicular to the tangent and normal vector is the unit binormal

$$\vec{B}(t) = \vec{T}(t)\times\vec{N}(t)$$

and the three orthogonal normalized (that is orthonormal) vectors $\vec{T}$, $\vec{N}(t)$ and $\vec{B}(t)$ form an natural system of coordinates for studying the curve.

Curvature: The curvature is a measure of how fast a curve turns. It is the reciprocal of the radius of the "osculating" circle. The curvature of the curve $\vec{r}$ at $\vec{r}(t)$ is

$$\kappa(t) = \frac{|\vec{v}(t)\times\vec{a}(t)|}{|\vec{v}|^3}$$

Example: Consider $\vec{r} = \cos 2\pi t\vec{i} + \sin 2\pi t\vec{j} + t\vec{k}$. Then

$$\begin{aligned}
\vec{v}(t) &= -2\pi\sin 2\pi t\vec{i} + 2\pi\cos 2\pi t\vec{j} + \vec{j} \\
\vec{T}(t) &= \frac{\vec{v}(t)}{\sqrt{4\pi^2+1}} \\
\vec{a}(t) &= -4\pi^2\cos 2\pi t\vec{i} - 4\pi^2\sin 2\pi t \\
\vec{N}(t) &= -\cos 2\pi t\vec{i} - \sin 2\pi t \\
\vec{B}(t) &= \frac{1}{\sqrt{4\pi^2+1}}\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2\pi\sin 2\pi t & 2\pi\cos 2\pi t & 1 \\ -\cos 2\pi t & -\sin 2\pi t & 0 \end{bmatrix} \\
&= \frac{1}{\sqrt{4\pi^2+1}}\left[\sin 2\pi t\vec{i} - \cos 2\pi t\vec{j} + 2\pi\vec{k}\right] \\
\kappa &= \frac{|\vec{v}\times\vec{a}|}{|\vec{v}(t)|^3} \\
&= \frac{4\pi^2\sqrt{4\pi^2+1}}{(\sqrt{4\pi^2+1})^3} = \frac{4\pi^2}{4\pi^2+1}
\end{aligned}$$