15. Double Integrals and Area: The area of a region R in the xy–plane is same as the volume of the solid that sits above R and is 1 unit thick.

$$\text{Area } = \iint_R dA = \iint_R \mathbf{1}\, dA$$

where $\mathbf{1}$ is the identically one function.

Example: Find the area of the region bounded by the curves $x = y^2 - 2y$ and $x = 2y - y^2$.

The curves intersect when $y^2 - 2y = 2y - y^2$ or $2y^2 - 4y = 0$ or $2y(y-2) = 0$, that is $y = 0$ and $y = 2$.

The region is of type II. Sketch? The area is

$$\int_0^2 \int_{y^2-2y}^{2y-y^2} dx\, dy = \int_0^2 2y - y^2 - (y^2 - 2y)\, dy = \int_0^2 4y - 2y^2\, dy = 2y^2 - \frac{2}{3}y^3|_0^2 = \frac{8}{3}$$