LESSON 4 COTERMINAL ANGLES

Topics in this lesson:

1. THE DEFINITION AND EXAMPLES OF COTERMINAL ANGLES
2. FINDING COTERMINAL ANGLES
3. TRIGONOMETRIC FUNCTIONS OF COTERMINAL ANGLES

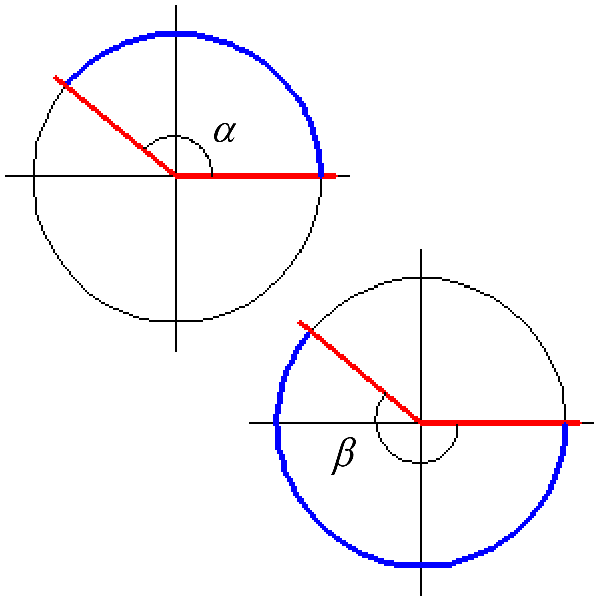
1. THE DEFINITION AND EXAMPLES OF COTERMINAL ANGLES

Definition Two angles are said to be coterminal if their terminal sides are the same.

Examples Here are some examples of coterminal angles.

1. The two angles of $\alpha = 140^\circ$ and $\beta = -\,220^\circ$ are coterminal angles.

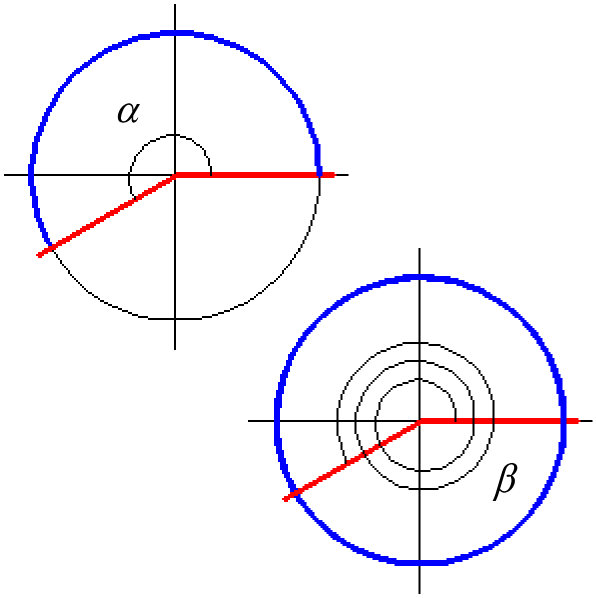
The Two Angles of 140 Degrees and Negative 220 Degrees Are Coterminal



Animation of the making of these two coterminal angles.

2. The two angles of $\alpha = \dfrac{7\pi}{6}$ and $\beta = \dfrac{31\pi}{6}$ are coterminal angles.

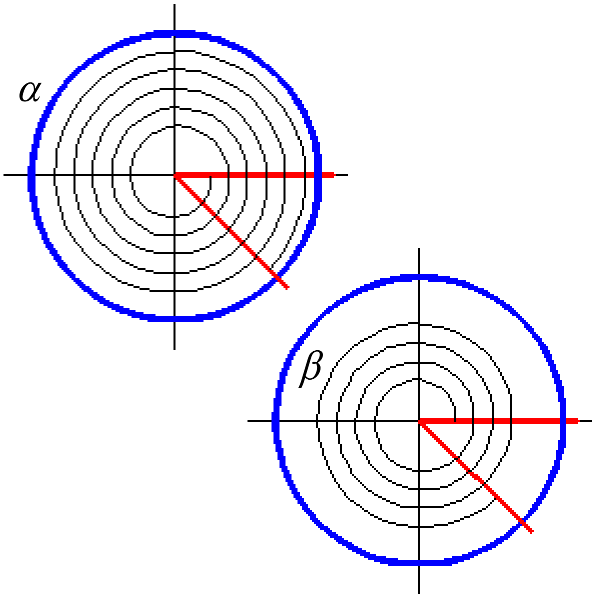
The Two Angles of 7Pi/6 and 31Pi/6 Are Coterminal



Animation of the making of these two coterminal angles.

3. The two angles of $\alpha = -1845^\circ$ and $\beta = 1395^\circ$ are coterminal angles.

The Two Angles of Negative 1845 Degrees and 1395 Degrees Are Coterminal



Animation of the making of these two coterminal angles.

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2. FINDING COTERMINAL ANGLES

Theorem The difference between two coterminal angles is a multiple (positive or negative) of 2π or $360°$.

Examples Find three positive and three negative angles that are coterminal with the following angles.

1. $\theta = 660°$

$\theta_1 = 660° + 360° = 1020°$

$\theta_2 = 1020° + 360° = 1380°$

NOTE: $\theta_2 = 1380° = 1020° + 360° = (660° + 360°) + 360° = 660° + 2(360°)$

$\theta_3 = 660° - 360° = 300°$

$\theta_4 = 300° - 360° = -60°$

NOTE: $\theta_4 = -60° = 300° - 360° = (660° - 360°) - 360° = 660° - 2(360°)$

$\theta_5 = -60° - 360° = -420°$

NOTE: $\theta_5 = -420° = -60° - 360° = [660° - 2(360°)] - 360° = 660° - 3(360°)$

$\theta_6 = -420° - 360° = -780°$

NOTE: $\theta_6 = -780° = -420° - 360° = [660° - 3(360°)] - 360° = 660° - 4(360°)$

NOTE: There are other answers for this problem. If the difference between the angle of $\theta = 660\,°$ and another angle is a multiple of $360\,°$, then this second angle is an answer to problem.

2. $\alpha = \dfrac{87\pi}{5}$

$$\alpha_1 = \frac{87\pi}{5} - 2\pi = \frac{87\pi}{5} - \frac{10\pi}{5} = \frac{77\pi}{5}$$

$$\alpha_2 = \frac{77\pi}{5} - 2\pi = \frac{77\pi}{5} - \frac{10\pi}{5} = \frac{67\pi}{5}$$

NOTE: $\alpha_2 = \dfrac{67\pi}{5} = \dfrac{77\pi}{5} - 2\pi = \left(\dfrac{87\pi}{5} - 2\pi\right) - 2\pi = \dfrac{87\pi}{5} - 2(2\pi)$

$$\alpha_3 = \frac{67\pi}{5} - 2\pi = \frac{67\pi}{5} - \frac{10\pi}{5} = \frac{57\pi}{5}$$

NOTE: $\alpha_3 = \dfrac{57\pi}{5} = \dfrac{67\pi}{5} - 2\pi = \left(\dfrac{87\pi}{5} - 2(2\pi)\right) - 2\pi = \dfrac{87\pi}{5} - 3(2\pi)$

$$\alpha_4 = \frac{87\pi}{5} - 9(2\pi) = \frac{87\pi}{5} - 9\left(\frac{10\pi}{5}\right) = \frac{87\pi}{5} - \frac{90\pi}{5} = -\frac{3\pi}{5}$$

$$\alpha_5 = -\frac{3\pi}{5} - 2\pi = -\frac{3\pi}{5} - \frac{10\pi}{5} = -\frac{13\pi}{5}$$

NOTE: $\alpha_5 = -\dfrac{13\pi}{5} = -\dfrac{3\pi}{5} - 2\pi = \left(\dfrac{87\pi}{5} - 9(2\pi)\right) - 2\pi = \dfrac{87\pi}{5} - 10(2\pi)$

$$\alpha_6 = -\frac{13\pi}{5} - 2\pi = -\frac{13\pi}{5} - \frac{10\pi}{5} = -\frac{23\pi}{5}$$

NOTE: $\alpha_6 = -\dfrac{23\pi}{5} = -\dfrac{13\pi}{5} - 2\pi = \left(\dfrac{87\pi}{5} - 10(2\pi)\right) - 2\pi = \dfrac{87\pi}{5} - 11(2\pi)$

NOTE: There are also other answers for this problem. If the difference between the angle of $\alpha = \dfrac{87\pi}{5}$ and another angle is a multiple of 2π, then this second angle is an answer to problem.

Let’s see how the angle $\alpha = \dfrac{87\pi}{5}$ is made. We will need the following property of arithmetic, which comes from the check for long division.

$$\begin{array}{r} q \\ b\,\overline{)\;a\;} \\ \underline{\quad\;} \\ r \end{array} \quad \Rightarrow \quad a = q \cdot b + r$$

Dividing both sides of the equation $a = q \cdot b + r$ by *b*, we obtain the equation $\dfrac{a}{b} = q + \dfrac{r}{b}$. For the work that we will do in order to find one particular coterminal angle of a given angle in radians, we will want the number *q* above to be an even number.

Now, consider the fraction of $\dfrac{87}{5}$ in the angle $\alpha = \dfrac{87\pi}{5}$ given above.

$$\frac{87}{5} \rightarrow \begin{array}{r} 17 \\ 5\,\overline{)\,87} \\ \underline{5} \\ 37 \\ \underline{35} \\ 2 \end{array}$$

Thus, $\dfrac{87}{5} = 17 + \dfrac{2}{5}$. The number 17 is an odd number. However, we may write 17 as $17 = 16 + 1$, where the number 16 is an even number. Thus, we have that

$$\frac{87}{5} = 17 + \frac{2}{5} = (16+1) + \frac{2}{5} = 16 + \left(1 + \frac{2}{5}\right) = 16 + \left(\frac{5}{5} + \frac{2}{5}\right) = 16 + \frac{7}{5}$$

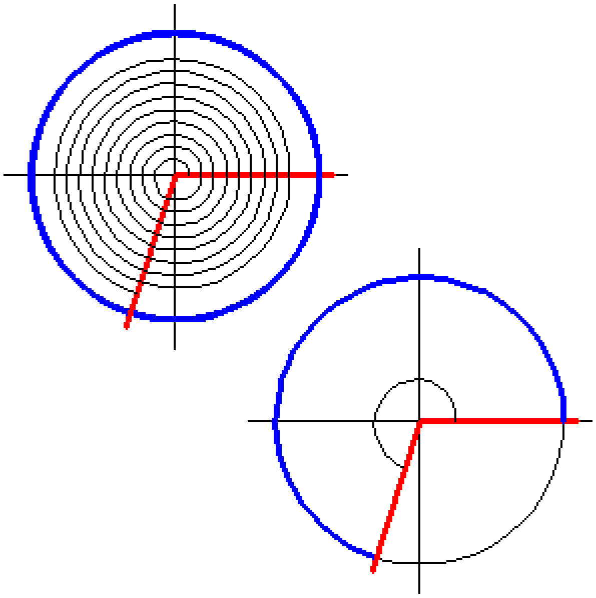
Thus, $\frac{87}{5} = 16 + \frac{7}{5}$. Now, multiply both sides of this equation by π to obtain that

$$\frac{87\pi}{5} = \pi\left(16 + \frac{7}{5}\right) = 16\pi + \frac{7\pi}{5} = 8(2\pi) + \frac{7\pi}{5}$$

This equation tells us how to make the angle $\alpha = \frac{87\pi}{5}$. In order to make the angle $\alpha = \frac{87\pi}{5}$, it will take eight complete revolutions and an additional rotation of $\frac{7\pi}{5}$ going in the counterclockwise direction in order to make the angle $\alpha = \frac{87\pi}{5}$.

Thus, the two angles $\alpha = \frac{87\pi}{5}$ and $\frac{7\pi}{5}$ are coterminal angles. Notice that $\frac{7\pi}{5} < 2\pi$.

The Angles of 87Pi/5 and 7Pi/5 Are Coterminal



Animation of the making of these two coterminal angles.

Another animation of the making of these two coterminal angles.

Examples Find the angle between 0 and 2π or the angle between -2π and 0 that is coterminal with the following angles.

1. $\theta = \dfrac{85\pi}{6}$

Consider the fraction of $\dfrac{85}{6}$ in the angle $\theta = \dfrac{85\pi}{6}$.

$$\frac{85}{6} \rightarrow \begin{array}{r} 14 \\ 6\,\overline{)\,85} \\ \underline{6} \\ 25 \\ \underline{24} \\ 1 \end{array}$$

Thus, $\dfrac{85}{6} = 14 + \dfrac{1}{6}$. The 14 is an even number. So, multiply both sides of this equation by π to obtain that

$$\frac{85\pi}{6} = \pi\left(14 + \frac{1}{6}\right) = 14\pi + \frac{\pi}{6} = 7(2\pi) + \frac{\pi}{6}$$

In order to make the angle $\theta = \dfrac{85\pi}{6}$, it will take seven complete revolutions and an additional rotation of $\dfrac{\pi}{6}$ going in the counterclockwise direction. Thus, the angle of $\dfrac{\pi}{6}$ is the coterminal angle that we are looking for.

Animation of the making of these two coterminal angles. Another animation of the making of these two coterminal angles.

Answer: $\frac{\pi}{6}$

2. $\beta = -\frac{91\pi}{4}$

Consider the fraction of $\frac{91}{4}$ in the angle $\beta = -\frac{91\pi}{4}$.

$$\frac{91}{4} \rightarrow \begin{array}{r} 22 \\ 4\overline{)91} \\ \underline{8} \\ 11 \\ \underline{8} \\ 3 \end{array}$$

Thus, $\frac{91}{4} = 22 + \frac{3}{4}$. The 22 is an even number. Now, multiply both sides of this equation by $-\pi$ to obtain that

$$-\frac{91\pi}{4} = -\pi\left(22 + \frac{3}{4}\right) = -22\pi - \frac{3\pi}{4} = 11(-2\pi) - \frac{3\pi}{4}$$

In order to make the angle $\beta = -\frac{91\pi}{4}$, it will take eleven complete revolutions and an additional rotation of $\frac{3\pi}{4}$ going in the **clockwise** direction. Thus, the angle of $-\frac{3\pi}{4}$ is the coterminal angle that we are looking for. Animation of the making of these two coterminal angles.

Answer: $-\dfrac{3\pi}{4}$

3. $\gamma = -\dfrac{112\ \pi}{3}$

Consider the fraction of $\dfrac{112}{3}$ in the angle $\gamma = -\dfrac{112\ \pi}{3}$.

$$\frac{112}{3} \quad \rightarrow \quad \begin{array}{r} 37 \\ 3\overline{)112} \\ \underline{9} \\ 22 \\ \underline{21} \\ 1 \end{array}$$

Thus, $\dfrac{112}{3} = 37 + \dfrac{1}{3}$. The 37 is an odd number. So, we may write 37 as $37 = 36 + 1$, where the number 36 is an even number. Thus, we have that

$$\frac{112}{3} = 37 + \frac{1}{3} = (36 + 1) + \frac{1}{3} = 36 + \left(1 + \frac{1}{3}\right) = 36 + \left(\frac{3}{3} + \frac{1}{3}\right) = 36 + \frac{4}{3}$$

Thus, $\dfrac{112}{3} = 36 + \dfrac{4}{3}$. Now, multiply both sides of this equation by $-\pi$ to obtain that

$$-\frac{112\ \pi}{3} = -\pi\left(36 + \frac{4}{3}\right) = -36\pi - \frac{4\pi}{3} = 18(-2\pi) - \frac{4\pi}{3}$$

In order to make the angle $\gamma = -\dfrac{112\,\pi}{3}$, it will take eighteen complete revolutions and an additional rotation of $\dfrac{4\pi}{3}$ going in the **clockwise** direction. Thus, the angle of $-\dfrac{4\pi}{3}$ is the coterminal angle that we are looking for.

Answer: $-\dfrac{4\pi}{3}$

4. $\alpha = \dfrac{59\,\pi}{2}$

Consider the fraction of $\dfrac{59}{2}$ in the angle $\alpha = \dfrac{59\,\pi}{2}$.

$$\frac{59}{2} \quad \rightarrow \quad \begin{array}{r} 29 \\ 2\,\overline{)\,59} \\ \underline{4} \\ 19 \\ \underline{18} \\ 1 \end{array}$$

Thus, $\dfrac{59}{2} = 29 + \dfrac{1}{2}$. The 29 is an odd number. So, we may write 29 as $29 = 28 + 1$, where the number 28 is an even number. Thus, we have that

$$\frac{59}{2} = 29 + \frac{1}{2} = (28 + 1) + \frac{1}{2} = 28 + \left(1 + \frac{1}{2}\right) = 28 + \left(\frac{2}{2} + \frac{1}{2}\right) = 28 + \frac{3}{2}$$

Thus, $\dfrac{59}{2} = 28 + \dfrac{3}{2}$. Now, multiply both sides of this equation by π to obtain that

$$\frac{59\pi}{2} = \pi\left(28 + \frac{3}{2}\right) = 28\pi + \frac{3\pi}{2} = 14(2\pi) + \frac{3\pi}{2}$$

In order to make the angle $\alpha = \frac{59\pi}{2}$, it will take fourteen complete revolutions and an additional rotation of $\frac{3\pi}{2}$ going in the counterclockwise direction. Thus, the angle of $\frac{3\pi}{2}$ is the coterminal angle that we are looking for. Animation of the making of these two coterminal angles.

Answer: $\frac{3\pi}{2}$

5. $\theta = -\frac{17\pi}{2}$

Consider the fraction of $\frac{17}{2}$ in the angle $\theta = -\frac{17\pi}{2}$.

$$\frac{17}{2} \rightarrow \begin{array}{r} 8 \\ 2\,\overline{)\,17} \\ \underline{16} \\ 1 \end{array}$$

Thus, $\frac{17}{2} = 8 + \frac{1}{2}$. The 8 is an even number. Now, multiply both sides of this equation by $-\pi$ to obtain that

$$-\frac{17\pi}{2} = -\pi\left(8 + \frac{1}{2}\right) = -8\pi - \frac{\pi}{2} = 4(-2\pi) - \frac{\pi}{2}$$

In order to make the angle $\theta = -\dfrac{17\pi}{2}$, it will take four complete revolutions and an additional rotation of $\dfrac{\pi}{2}$ going in the **clockwise** direction. Thus, the angle of $-\dfrac{\pi}{2}$ is the coterminal angle that we are looking for. Animation of the making of these two coterminal angles.

Answer: $-\dfrac{\pi}{2}$

6. $\phi = 82\pi$

The number 82 is an even number. Thus, $82\pi = 41(2\pi)$. In order to make the angle $\phi = 82\pi$, it will take forty-one complete revolutions. Thus, the angle of 0 is the coterminal angle that we are looking for.

Answer: 0

7. $\beta = -21\pi$

The 21 is an odd number. So, we may write 21 as $21 = 20 + 1$, where the number 20 is an even number. Now, multiply both sides of this equation by $-\pi$ to obtain that

$$-21\pi = -\pi(20+1) = -20\pi - \pi = 10(-2\pi) - \pi$$

In order to make the angle $\beta = -21\pi$, it will take ten complete revolutions and an additional rotation of π going in the **clockwise** direction. Thus, the angle of $-\pi$ is the coterminal angle that we are looking for.

Answer: $-\pi$

8. $\gamma = 4110^\circ$

Use your calculator to find the whole number of times that 360 will divide into 4110 . Since $4110 \div 360 = 11.41666$ .... and 360 times 11 equals 3960, then we have that

$$\begin{array}{r} 11 \\ 360\,\overline{)\,4110} \\ 3960 \\ \hline 150 \end{array}$$

Thus, $4110 = 11(360) + 150$. When working in degrees, the whole number of 11, in this case, does not have to be even. Thus,

$$4110 = 11(360) + 150 \;\Rightarrow\; 4110\,° = 11(360\,°) + 150\,°$$

In order to make the angle $\gamma = 4110\,°$, it will take eleven complete revolutions and an additional rotation of $150\,°$ going in the counterclockwise direction. Thus, the angle of $150\,°$ is the coterminal angle that we are looking for.

Answer: $150\,°$

9. $\theta = -\,8865\,°$

Use your calculator to find the whole number of times that 360 will divide into 8865 . Since $8865 \div 360 = 24.625$ and 360 times 24 equals 8640, then we have that

$$\begin{array}{r} 24 \\ 360\,\overline{)\,8865} \\ 8640 \\ \hline 225 \end{array}$$

Thus, $8865 = 24(360) + 225$. Multiplying both sides of this equation by -1, we obtain that $-8865 = 24(-360) - 225$. Thus,

$$-8865 = 24(-360) - 225 \Rightarrow -8865^{\circ} = 24(-360^{\circ}) - 225^{\circ}$$

In order to make the angle $\theta = -8865^{\circ}$, it will take twenty-four complete revolutions and an additional rotation of 225° going in the **clockwise** direction. Thus, the angle of -225° is the coterminal angle that we are looking for.

Answer: -225°

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3. TRIGONOMETRIC FUNCTIONS OF COTERMINAL ANGLES

Theorem Let α and β be two coterminal angles. Then

1. $\cos\alpha = \cos\beta$
2. $\sin\alpha = \sin\beta$
3. $\tan\alpha = \tan\beta$
4. $\sec\alpha = \sec\beta$
5. $\csc\alpha = \csc\beta$
6. $\cot\alpha = \cot\beta$

We can use this theorem to find any one of the six trigonometric functions of an angle that is numerically bigger than 2π or 360°. When given an angle that is numerically bigger than 2π or 360°, we will want to find the angle that is coterminal to it and is numerically smaller than 2π or 360°.

Examples Use a coterminal angle to find the exact value of the six trigonometric functions of the following angles.

1. $\theta = \dfrac{11\pi}{3}$

Doing one subtraction of 2π, we obtain that

$$\frac{11\pi}{3} - 2\pi = \frac{11\pi}{3} - \frac{6\pi}{3} = \frac{5\pi}{3}$$

Thus, the two angles of $\theta = \frac{11\pi}{3}$ and $\frac{5\pi}{3}$ are coterminal. The angle $\frac{5\pi}{3}$ is in the IV quadrant and has a reference angle of $\frac{\pi}{3}$. Thus, by the theorem above, we have that

$$\cos\left(\frac{11\pi}{3}\right) = \cos\left(\frac{5\pi}{3}\right) = \cos\frac{\pi}{3} = \frac{1}{2} \qquad \sec\left(\frac{11\pi}{3}\right) = 2$$

$$\sin\left(\frac{11\pi}{3}\right) = \sin\left(\frac{5\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2} \qquad \csc\left(\frac{11\pi}{3}\right) = -\frac{2}{\sqrt{3}}$$

$$\tan\left(\frac{11\pi}{3}\right) = \tan\left(\frac{5\pi}{3}\right) = -\tan\frac{\pi}{3} = -\sqrt{3} \qquad \cot\left(\frac{11\pi}{3}\right) = -\frac{1}{\sqrt{3}}$$

2. $\alpha = -\frac{19\pi}{4}$

Doing one addition of 2π, we obtain that

$$-\frac{19\pi}{4} + 2\pi = -\frac{19\pi}{4} + \frac{8\pi}{4} = -\frac{11\pi}{4}$$

Doing a second addition of 2π, we obtain that

$$-\frac{11\pi}{4} + 2\pi = -\frac{11\pi}{4} + \frac{8\pi}{4} = -\frac{3\pi}{4}$$

Thus, the two angles of $\alpha = -\frac{19\,\pi}{4}$ and $-\frac{3\,\pi}{4}$ are coterminal. The angle $-\frac{3\pi}{4}$ is in the III quadrant and has a reference angle of $\frac{\pi}{4}$. Thus, by the theorem above, we have that

$$\cos\left(-\frac{19\pi}{4}\right) = \cos\left(-\frac{3\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2} \qquad \sec\left(-\frac{19\pi}{4}\right) = -\sqrt{2}$$

$$\sin\left(-\frac{19\pi}{4}\right) = \sin\left(-\frac{3\pi}{4}\right) = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2} \qquad \csc\left(-\frac{19\pi}{4}\right) = -\sqrt{2}$$

$$\tan\left(-\frac{19\pi}{4}\right) = \tan\left(-\frac{3\pi}{4}\right) = \tan\frac{\pi}{4} = 1 \qquad \cot\left(-\frac{19\pi}{4}\right) = 1$$

3. $\beta = \frac{149\,\pi}{6}$

Consider the fraction of $\frac{149}{6}$ in the angle $\beta = \frac{149\,\pi}{6}$.

$$\frac{149}{6} \quad \to \quad \begin{array}{r} 24 \\ 6\,\big)\overline{\,149} \\ \underline{12} \\ 29 \\ \underline{24} \\ 5 \end{array}$$

Thus, $\frac{149}{6} = 24 + \frac{5}{6}$. This implies that

$$\frac{149\,\pi}{6} = 24\pi + \frac{5\pi}{6} = 12\,(2\pi) + \frac{5\pi}{6}$$

In order to make the angle $\beta = \frac{149\pi}{6}$, it will take twelve complete revolutions and an additional rotation of $\frac{5\pi}{6}$ going in the counterclockwise direction. Thus, the angle of $\frac{5\pi}{6}$ is the coterminal angle that we are looking for. The angle $\frac{5\pi}{6}$ is in the II quadrant and has a reference angle of $\frac{\pi}{6}$. Thus, by the theorem above, we have that

$$\cos\frac{149\pi}{6} = \cos\frac{5\pi}{6} = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2} \qquad \sec\frac{149\pi}{6} = -\frac{2}{\sqrt{3}}$$

$$\sin\frac{149\pi}{6} = \sin\frac{5\pi}{6} = \sin\frac{\pi}{6} = \frac{1}{2} \qquad \csc\frac{149\pi}{6} = 2$$

$$\tan\frac{149\pi}{6} = \tan\frac{5\pi}{6} = -\tan\frac{\pi}{6} = -\frac{1}{\sqrt{3}} \qquad \cot\frac{149\pi}{6} = -\sqrt{3}$$

4. $\gamma = \frac{58\pi}{3}$

Consider the fraction of $\frac{58}{3}$ in the angle $\gamma = \frac{58\pi}{3}$.

$$\frac{58}{3} \to \begin{array}{r} 19 \\ 3\,\overline{)\,58} \\ \underline{3} \\ 28 \\ \underline{27} \\ 1 \end{array}$$

Thus, $\frac{58}{3} = 19 + \frac{1}{3} = 18 + \frac{4}{3}$. This implies that

$$\frac{58\pi}{3} = 18\pi + \frac{4\pi}{3} = 9(2\pi) + \frac{4\pi}{3}$$

In order to make the angle $\gamma = \frac{58\pi}{3}$, it will take nine complete revolutions and an additional rotation of $\frac{4\pi}{3}$ going in the counterclockwise direction. Thus, the angle of $\frac{4\pi}{3}$ is the coterminal angle that we are looking for. The angle $\frac{4\pi}{3}$ is in the III quadrant and has a reference angle of $\frac{\pi}{3}$. Thus, by the theorem above, we have that

$$\cos\frac{58\pi}{3} = \cos\frac{4\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2} \qquad \sec\frac{58\pi}{3} = -2$$

$$\sin\frac{58\pi}{3} = \sin\frac{4\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2} \qquad \csc\frac{58\pi}{3} = -\frac{2}{\sqrt{3}}$$

$$\tan\frac{58\pi}{3} = \tan\frac{4\pi}{3} = \tan\frac{\pi}{3} = \sqrt{3} \qquad \cot\frac{58\pi}{3} = \frac{1}{\sqrt{3}}$$

5. $\phi = -\frac{169\pi}{4}$

Consider the fraction of $\frac{169}{4}$ in the angle $\phi = -\frac{169\pi}{4}$.

$$\frac{169}{4} \quad \rightarrow \quad \begin{array}{r} 42 \\ 4\,\overline{)\,169} \\ \underline{16} \\ 9 \\ \underline{8} \\ 1 \end{array}$$

Thus, $\frac{169}{4} = 42 + \frac{1}{4}$. This implies that

$$-\frac{169\pi}{4} = -42\pi - \frac{\pi}{4} = 21(-2\pi) - \frac{\pi}{4}$$

In order to make the angle $\phi = -\frac{169\pi}{4}$, it will take twenty-one complete revolutions and an additional rotation of $\frac{\pi}{4}$ going in the **clockwise** direction. Thus, the angle of $-\frac{\pi}{4}$ is the coterminal angle that we are looking for. The angle $-\frac{\pi}{4}$ is in the IV quadrant and has a reference angle of $\frac{\pi}{4}$. Thus, by the theorem above, we have that

$$\cos\left(-\frac{169\pi}{4}\right) = \cos\left(-\frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \sec\left(-\frac{169\pi}{4}\right) = \sqrt{2}$$

$$\sin\left(-\frac{169\pi}{4}\right) = \sin\left(-\frac{\pi}{4}\right) = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2} \qquad \csc\left(-\frac{169\pi}{4}\right) = -\sqrt{2}$$

$$\tan\left(-\frac{169\pi}{4}\right) = \tan\left(-\frac{\pi}{4}\right) = -\tan\frac{\pi}{4} = -1 \qquad \cot\left(-\frac{169\pi}{4}\right) = -1$$

6. $\theta = -\dfrac{527\ \pi}{6}$

Consider the fraction of $\dfrac{527}{6}$ in the angle $\theta = -\dfrac{527\ \pi}{6}$.

$$\frac{527}{6} \rightarrow \begin{array}{r} 87 \\ 6\,\overline{)\,527} \\ \underline{48} \\ 47 \\ \underline{42} \\ 5 \end{array}$$

Thus, $\dfrac{527}{6} = 87 + \dfrac{5}{6} = 86 + \dfrac{11}{6}$. This implies that

$$-\frac{527\ \pi}{6} = -86\pi - \frac{11\ \pi}{6} = 43(-2\pi) - \frac{11\ \pi}{6}$$

In order to make the angle $\theta = -\dfrac{527\ \pi}{6}$, it will take forty-three complete revolutions and an additional rotation of $\dfrac{11\ \pi}{6}$ going in the **clockwise** direction. Thus, the angle of $-\dfrac{11\ \pi}{6}$ is the coterminal angle that we are looking for. The angle $-\dfrac{11\ \pi}{6}$ is in the I quadrant and has a reference angle of $\dfrac{\pi}{6}$. Thus, by the theorem above, we have that

$$\cos\left(-\frac{527\pi}{6}\right) = \cos\left(-\frac{11\ \pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \qquad \sec\left(-\frac{527\pi}{6}\right) = \frac{2}{\sqrt{3}}$$

$\sin\left(-\frac{527\pi}{6}\right) = \sin\left(-\frac{11\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$ $\quad\quad$ $\csc\left(-\frac{527\pi}{6}\right) = 2$

$\tan\left(-\frac{527\pi}{6}\right) = \tan\left(-\frac{11\pi}{6}\right) = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$ $\quad\quad$ $\cot\left(-\frac{527\pi}{6}\right) = \sqrt{3}$

7. $\alpha = -\frac{17\pi}{2}$

Consider the fraction of $\frac{17}{2}$ in the angle $\alpha = -\frac{17\pi}{2}$. Since $\frac{17}{2} = 8 + \frac{1}{2}$, then

$$-\frac{17\pi}{2} = -8\pi - \frac{\pi}{2} = 4(-2\pi) - \frac{\pi}{2}$$

In order to make the angle $\alpha = -\frac{17\pi}{2}$, it will take four complete revolutions and an additional rotation of $\frac{\pi}{2}$ going in the **clockwise** direction. Thus, the angle of $-\frac{\pi}{2}$ is the coterminal angle that we are looking for. The angle $-\frac{\pi}{2}$ lies on the negative *y*-axis. Thus, by the theorem above, we have that

$\cos\left(-\frac{17\pi}{2}\right) = \cos\left(-\frac{\pi}{2}\right) = 0$ $\quad\quad$ $\sec\left(-\frac{17\pi}{2}\right) =$ undefined

$\sin\left(-\frac{17\pi}{2}\right) = \sin\left(-\frac{\pi}{2}\right) = -1$ $\quad\quad$ $\csc\left(-\frac{17\pi}{2}\right) = -1$

$\tan\left(-\frac{17\pi}{2}\right) = \tan\left(-\frac{\pi}{2}\right) = \frac{-1}{0}$ = undefined $\quad\quad$ $\cot\left(-\frac{17\pi}{2}\right) = \frac{0}{-1} = 0$

8. $\beta = 65\pi$

Since $65\pi = 64\pi + \pi = 32(2\pi) + \pi$, then in order to make the angle $\beta = 65\pi$, it will take thirty-two complete revolutions and an additional rotation of π going in the counterclockwise direction. Thus, the angle of π is the coterminal angle that we are looking for. The angle π lies on the negative *x*-axis. Thus, by the theorem above, we have that

$\cos 65\pi = \cos \pi = -1$ $\qquad$ $\sec 65\pi = -1$

$\sin 65\pi = \sin \pi = 0$ $\qquad$ $\csc 65\pi =$ undefined

$\tan 65\pi = \tan \pi = \dfrac{0}{-1} = 0$ $\qquad$ $\cot 65\pi =$ undefined

9. $\gamma = -30\pi$

Since $-30\pi = 15(-2\pi)$, then in order to make the angle $\gamma = -30\pi$, it will take fifteen complete revolutions going in the **clockwise** direction. Thus, the angle of 0 is the coterminal angle that we are looking for. The angle 0 lies on the positive *x*-axis. Thus, by the theorem above, we have that

$\cos(-30\pi) = \cos 0 = 1$ $\qquad$ $\sec(-30\pi) = 1$

$\sin(-30\pi) = \sin 0 = 0$ $\qquad$ $\csc(-30\pi) =$ undefined

$\tan(-30\pi) = \tan 0 = \dfrac{0}{1} = 0$ $\qquad$ $\cot(-30\pi) =$ undefined

10. $\phi = 9840°$

Use your calculator to find the whole number of times that 360 will divide into 9840 . Since $9840 \div 360 = 27.3333$ .... and 360 times 27 equals 9720, then we have that

$$\begin{array}{r} 27 \\ 360 \overline{)\,9840} \\ \underline{9720} \\ 120 \end{array}$$

Thus, $9840 = 27(360) + 120$. Thus,

$$9840 = 27(360) + 120 \;\Rightarrow\; 9840\,^{\circ} = 27(360\,^{\circ}) + 120\,^{\circ}$$

In order to make the angle $\phi = 9840\,^{\circ}$, it will take twenty-seven complete revolutions and an additional rotation of $120\,^{\circ}$ going in the counterclockwise direction. Thus, the angle of $120\,^{\circ}$ is the coterminal angle that we are looking for. The angle $120\,^{\circ}$ is in the II quadrant and has a reference angle of $60\,^{\circ}$. Thus, by the theorem above, we have that

$$\cos 9840\,^{\circ} = \cos 120\,^{\circ} = -\cos 60\,^{\circ} = -\frac{1}{2} \qquad \sec 9840\,^{\circ} = -2$$

$$\sin 9840\,^{\circ} = \sin 120\,^{\circ} = \sin 60\,^{\circ} = \frac{\sqrt{3}}{2} \qquad \csc 9840\,^{\circ} = \frac{2}{\sqrt{3}}$$

$$\tan 9840\,^{\circ} = \tan 120\,^{\circ} = -\tan 60\,^{\circ} = -\sqrt{3} \qquad \cot 9840\,^{\circ} = -\frac{1}{\sqrt{3}}$$

11. $\alpha = 900^{\circ}$

Subtracting two times 360°, we obtain that $900^{\circ} - 720^{\circ} = 180^{\circ}$.

Thus, the two angles of $\alpha = 900^{\circ}$ and 180° are coterminal. The angle 180° is on the negative *x*-axis. Thus, using Unit Circle Trigonometry, we have that

$$\cos 900^{\circ} = \cos 180^{\circ} = -1 \qquad \sec 900^{\circ} = -1$$

$\sin 900° = \sin 180° = 0$ $\csc 900° =$ undefined

$\tan 900° = \tan 180° = 0$ $\cot 900° =$ undefined

12. $\beta = -1290°$

Adding three times $360°$, we obtain that $-1290° + 1080° = -210°$.

Thus, the two angles of $\beta = -1290°$ and $-210°$ are coterminal. The angle $-210°$ is in the II quadrant and has a reference angle of $30°$. Thus, by the theorem above, we have that

$$\cos(-1290°) = \cos(-210°) = -\cos 30° = -\frac{\sqrt{3}}{2}$$

$$\sec(-1290°) = -\frac{2}{\sqrt{3}}$$

$$\sin(-1290°) = \sin(-210°) = \sin 30° = \frac{1}{2} \qquad \csc(-1290°) = 2$$

$$\tan(-1290°) = \tan(-210°) = -\tan 30° = -\frac{1}{\sqrt{3}}$$

$$\cot(-1290°) = -\sqrt{3}$$

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