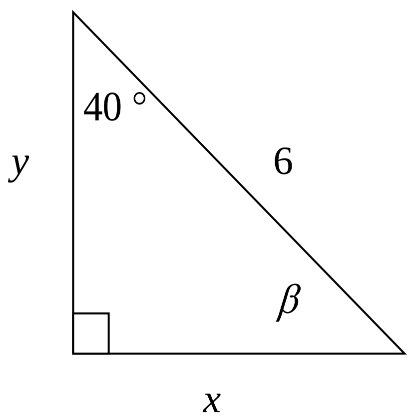
LESSON 7 SOLVING RIGHT TRIANGLES AND APPLICATIONS INVOLVING RIGHT TRIANGLES

Topics in this lesson:

1. SOLVING RIGHT TRIANGLES
2. APPLICATION PROBLEMS

1. SOLVING RIGHT TRIANGLES

Example Solve for x, y, and β.



To solve for β: Since the three angles of any triangle sum to $180°$, we get the following equation to solve.

$$\beta + 40° + 90° = 180° \Rightarrow \beta + 40° = 90° \Rightarrow \beta = 50°$$

Recall: Two angles that sum to $90°$ are called complimentary angles. The two acute angles in a right triangle are complimentary angles.

To solve for x: Notice in the right triangle, x is the opposite side of the given $40°$ angle and the given value of 6 is the hypotenuse of the right triangle. Restricting to the cosine, sine, and tangent functions, which one of these three functions involves the opposite side of the angle and the hypotenuse? Answer: The sine function. Thus, we have that

$$\frac{x}{6} = \sin 40° \Rightarrow x = 6\sin 40° \Rightarrow x \approx 3.86$$

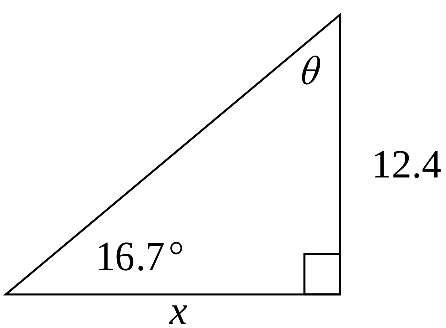
NOTE: Using your calculator, we have that $\sin 40° \approx 0.6427876097$.

To solve for y: Notice in the right triangle, y is the adjacent side of the given $40°$ angle and the given value of 6 is the hypotenuse of the right triangle. Restricting to the cosine, sine, and tangent functions, which one of these three functions involves the adjacent side of the angle and the hypotenuse? Answer: The cosine function. Thus, we have that

$$\frac{y}{6} = \cos 40° \Rightarrow \quad y = 6\cos 40° \Rightarrow y \approx 4.60$$

NOTE: Using your calculator, we have that $\cos 40° \approx 0.7660444431$.

<u>Example</u> Solve for x and θ.



To solve for θ: $\quad \theta + 16.7° = 90° \Rightarrow \theta = 73.3°$

To solve for x: Notice in the right triangle, x is the adjacent side of the given $16.7°$ angle and the given value of 12.4 is the opposite side of the given angle $16.7°$. Restricting to the cosine, sine, and tangent functions, which one of these three functions involves the opposite and adjacent sides of the angle? Answer: The tangent function. Thus, we have that

$$\frac{12.4}{x} = \tan 16.7° \Rightarrow \quad \frac{x}{12.4} = \frac{1}{\tan 16.7°} \Rightarrow \quad x = \frac{12.4}{\tan 16.7°} \approx 41.33 \quad \textbf{OR}$$

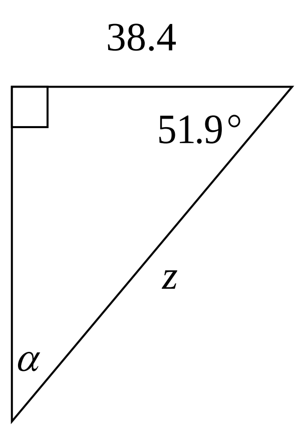
$$\frac{12.4}{x} = \tan 16.7° \Rightarrow \quad \frac{x}{12.4} = \cot 16.7° \Rightarrow \quad x = 12.4 \cot 16.7° \approx 41.33$$

NOTE: Using your calculator, we have that $\tan 16.7° \approx 0.3000143778$ and $\frac{1}{\tan 16.7°} = \cot 16.7° \approx 3.333173587$

Some students think that they use the secondary key that's with the $\boxed{\text{TAN}}$ key in order to find the cotangent of an angle. This is **NOT** correct. The (secondary) TAN^{-1} key is used to find the inverse tangent of a number. We will study the inverse trigonometric functions in Lesson 9.

In order to find the cotangent of an angle using your calculator, you use the $\boxed{\text{TAN}}$ key and the $\boxed{x^{-1}}$ key or the $\boxed{1/x}$ key.

Example Solve for z and α.



To solve for α: $\alpha + 51.9° = 90° \Rightarrow \alpha = 38.1°$

To solve for z: Notice in the right triangle, z is the hypotenuse of the right triangle and the given value of 38.4 is the adjacent side of the given angle $51.9°$. Restricting to the cosine, sine, and tangent functions, which one of these three functions involves the adjacent side of the angle and the hypotenuse? Answer: The cosine function. Thus, we have that

$$\frac{38.4}{z} = \cos 51.9° \Rightarrow \frac{z}{38.4} = \frac{1}{\cos 51.9°} \Rightarrow z = \frac{38.4}{\cos 51.9°} \approx 62.23$$ **OR**

$$\frac{38.4}{z} = \cos 51.9^\circ \Rightarrow \frac{z}{38.4} = \sec 51.9^\circ \Rightarrow z = 38.4 \sec 51.9^\circ \approx 62.23$$

NOTE: Using your calculator, we have that $\cos 51.9^\circ \approx 0.6170358751$ and

$$\frac{1}{\cos 51.9^\circ} = \sec 51.9^\circ \approx 1.620651311$$

In order to find the secant of an angle using your calculator, you use the COS key and the x^{-1} key or the $1/x$ key.

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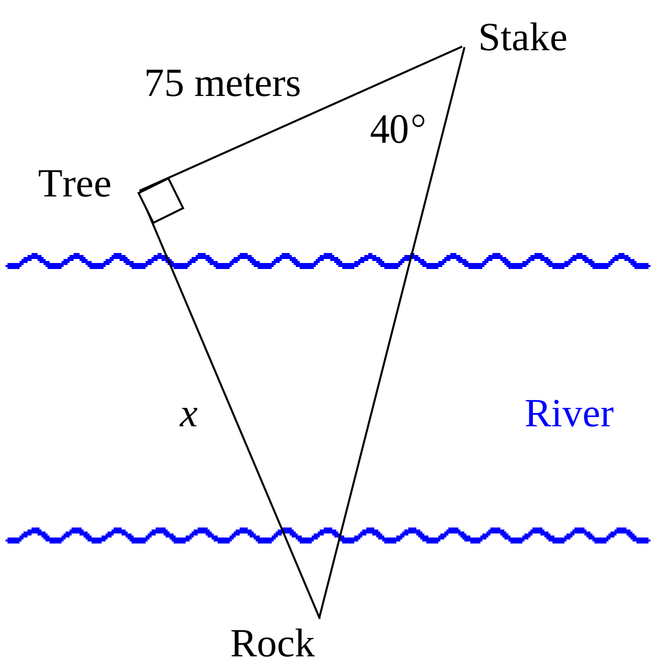
2. APPLICATION PROBLEMS

Examples Solve the following problems. Round your answers to the nearest hundredth. A diagram may be used to identify any variable(s).

1. A surveyor wishes to determine the distance between a rock and a tree on opposite sides of a river. He places a stake 75 meters from the tree so that a right triangle is formed by the stake, tree, and rock with the right angle at the tree. If the angle at the stake is 40°, what is the distance between the rock and the tree?



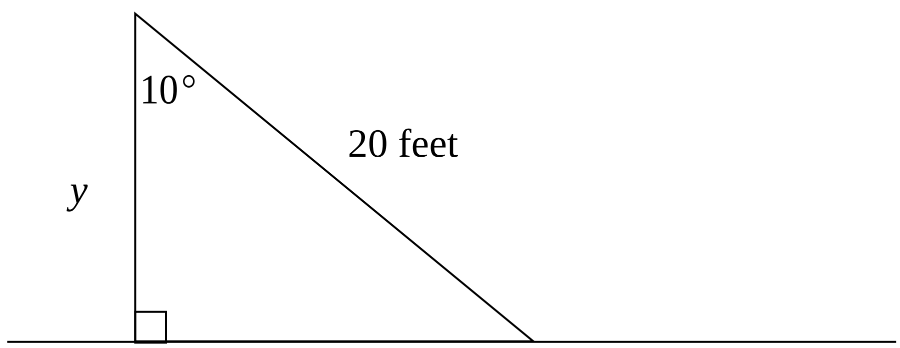
Notice in the right triangle, x is the opposite side of the given $40°$ angle and the given value of 75 meters is the adjacent side of the given angle $40°$. Thus,

$$\frac{x}{75} = \tan 40° \Rightarrow \quad x = 75 \tan 40° \approx 62.93$$

NOTE: $\tan 40° \approx 0.8390996312$

Answer: 62.93 meters

2. A 20-foot ladder is leaning against the top of a vertical wall. If the ladder makes an angle of $10°$ with the wall, how high is the wall?



Notice in the right triangle, y is the adjacent side of the given $10°$ angle and the given value of 20 feet is the hypotenuse of the right triangle. Thus,

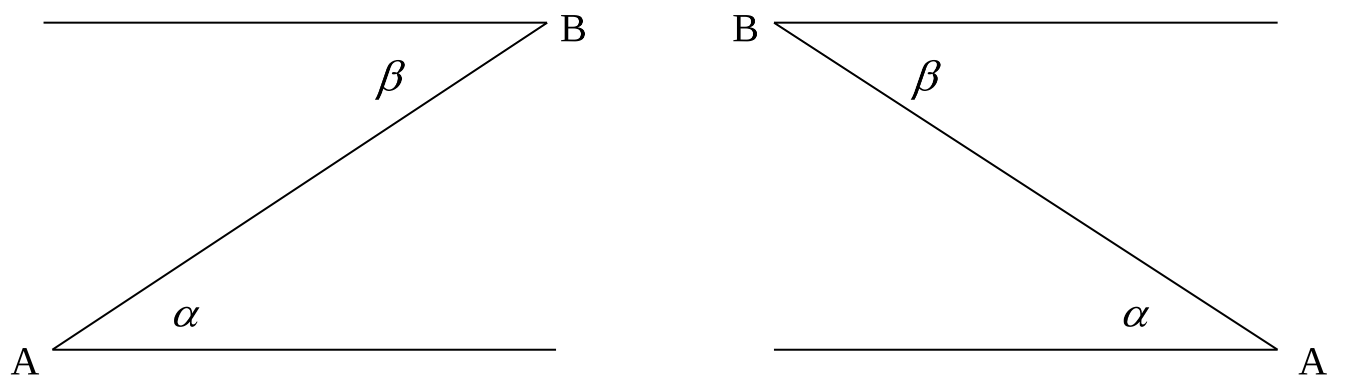
$$\frac{y}{20} = \cos 10° \Rightarrow \quad y = 20 \cos 10° \approx 19.70$$

NOTE: $\cos 10° \approx 0.984807753$

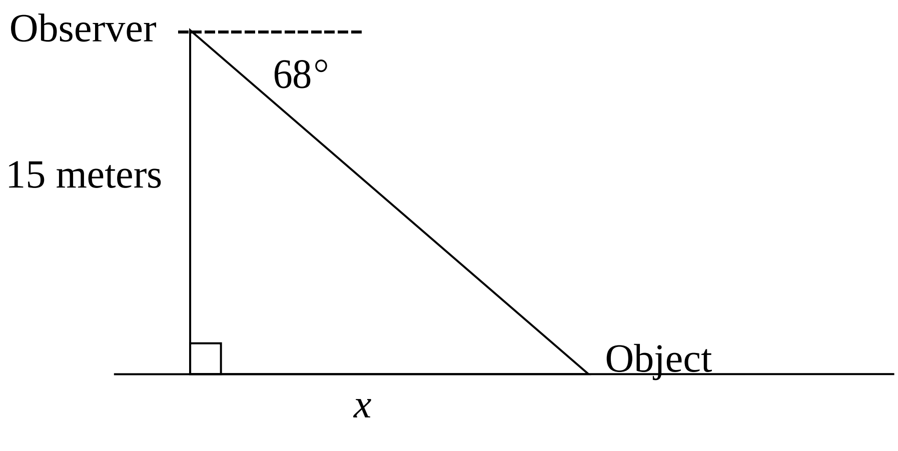
Answer: 19.70 feet

For the remaining examples, you will need the definition for **angle of elevation** and for **angle of depression**.

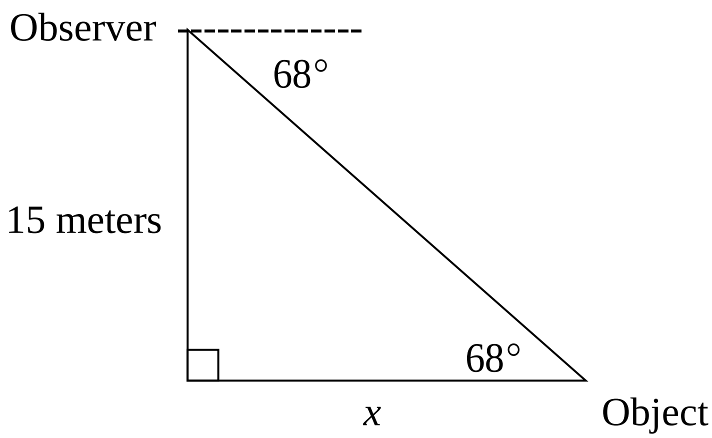
An angle of elevation and an angle of depression are both acute angles measured with respect to the horizontal. An angle of elevation is measured upward and an angle of depression is measured downward. The angle α below is an angle of elevation from the point A to the point B above. The angle β below is an angle of depression from the point B to the point A below.



3. From a point 15 meters above level ground, an observer measures the angle of depression of an object on the ground to be $68°$. How far is the object from the point on the ground directly beneath the observer?



NOTE: The angle of depression of $68°$ is an angle outside the right triangle. There are two ways to get an angle inside the triangle. The first way is to use alternating interior angles from geometry since we have two parallel lines being cut by a transversal.



Notice in the right triangle, x is the adjacent side of the given $68°$ angle and the given value of 15 meters is the opposite side of the given $68°$ angle. Thus,

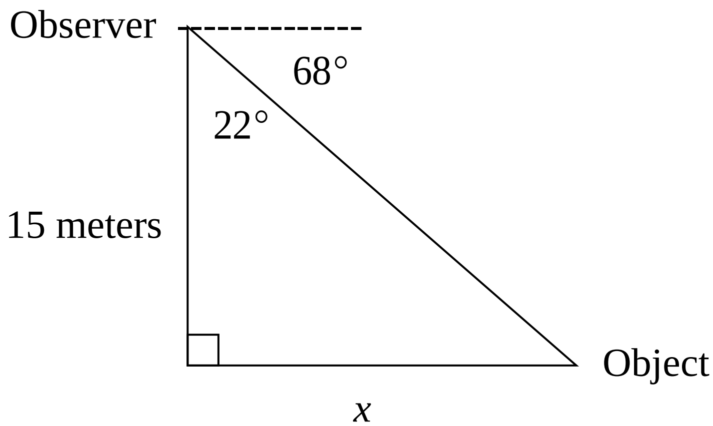
$$\frac{15}{x} = \tan 68° \Rightarrow \frac{x}{15} = \frac{1}{\tan 68°} \Rightarrow x = \frac{15}{\tan 68°} \approx 6.06$$ **OR**

$$\frac{15}{x} = \tan 68° \Rightarrow \frac{x}{15} = \cot 68° \Rightarrow x = 15 \cot 68° \approx 6.06$$

NOTE: $\tan 68° \approx 2.475086853$ and

$$\frac{1}{\tan 68°} = \cot 68° \approx 0.4040262258$$

The second way to get an angle inside the triangle is to notice that a right angle is formed at the observer by the horizontal and vertical lines. The angle of depression is using $68°$ of these $90°$. Thus, the angle inside the triangle at the observer is $22°$ obtained by $90° - 68°$.



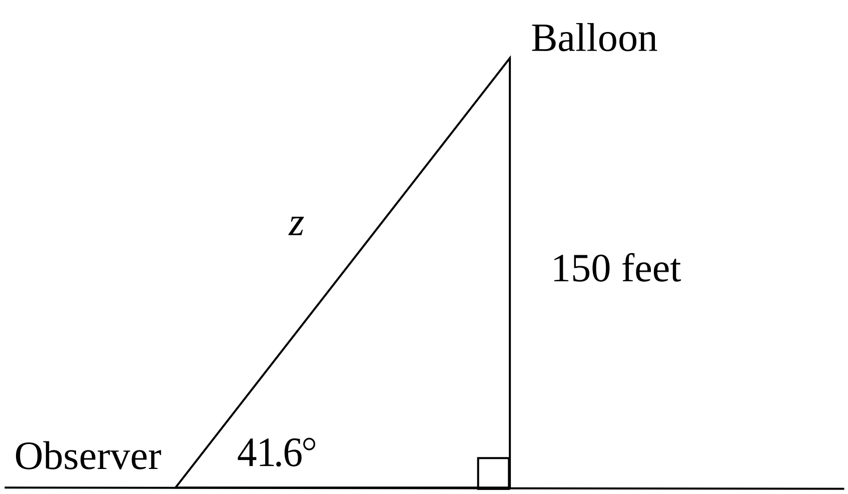
Notice in the right triangle, x is the opposite side of the given $22°$ angle and the given value of 15 meters is the adjacent side of the given $22°$ angle. Thus,

$$\frac{x}{15} = \tan 22° \Rightarrow \quad x = 15 \tan 22° \approx 6.06$$

NOTE: $\tan 22° \approx 0.4040262258$

Answer: 6.06 meters

4. A balloon is 150 feet above the ground. The angle of elevation from an observer on the ground to the balloon is $41.6°$. Find the distance from the observer to the balloon.



Notice in the right triangle, z is the hypotenuse of the right triangle and the given value of 150 feet is the opposite side of the given angle $41.6°$. Thus,

$$\frac{150}{z} = \sin 41.6° \Rightarrow \frac{z}{150} = \frac{1}{\sin 41.6°} \Rightarrow z = \frac{150}{\sin 41.6°} \approx 225.93$$ **OR**

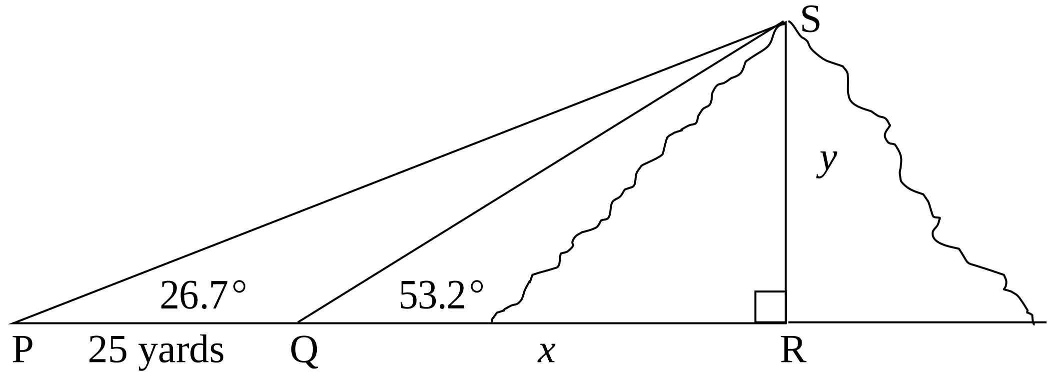
$$\frac{150}{z} = \sin 41.6° \Rightarrow \frac{z}{150} = \csc 41.6° \Rightarrow z = 150 \csc 41.6° \approx 225.93$$

NOTE: $\sin 41.6° \approx 0.6639262127$ and

$$\frac{1}{\sin 41.6°} = \csc 41.6° \approx 1.506191473$$

Answer: 225.93 feet

5. From a point P on level ground, the angle of elevation to the top of a mountain is $26.7°$. From a point 25 yards closer to the mountain and on the same line with P and the base of the mountain, the angle of elevation to the top of the mountain is $53.2°$. Find the height of the mountain.



Using Δ QRS, we have that $\frac{y}{x} = \tan 53.2°$

Using Δ PRS, we have that $\frac{y}{x + 25} = \tan 26.7°$

Since we want to find the height of the mountain, which is represented by y, then we want to solve this system of equations for y.

Use the first equation to solve for x in terms of y:

$$\frac{y}{x} = \tan 53.2^\circ \Rightarrow \frac{x}{y} = \cot 53.2^\circ \Rightarrow x = y \cot 53.2^\circ$$

Now, get rid of the fraction in the second equation by multiplying both sides of the equation by $x + 25$:

$$\frac{y}{x + 25} = \tan 26.7^\circ \Rightarrow y = (x + 25) \tan 26.7^\circ$$

Now, substitute $y \cot 53.2^\circ$ for x in the last equation:

$$y = (x + 25) \tan 26.7^\circ \Rightarrow y = (y \cot 53.2^\circ + 25) \tan 26.7^\circ$$

Now, distribute $\tan 26.7^\circ$ through the parentheses on the right side of the last equation:

$$y = (y \cot 53.2^\circ + 25) \tan 26.7^\circ \Rightarrow$$

$$y = y \cot 53.2^\circ \tan 26.7^\circ + 25 \tan 26.7^\circ$$

Now, put all the terms containing y on the left side of the equation. Thus, subtract $y \cot 53.2^\circ \tan 26.7^\circ$ from both sides of the last equation:

$$y = y \cot 53.2^\circ \tan 26.7^\circ + 25 \tan 26.7^\circ \Rightarrow$$

$$y - y \cot 53.2^\circ \tan 26.7^\circ = 25 \tan 26.7^\circ$$

Now, factor out the common y on the left side of the last equation:

$$y - y \cot 53.2^\circ \tan 26.7^\circ = 25 \tan 26.7^\circ \Rightarrow$$

$$y(1 - \cot 53.2° \tan 26.7°) = 25 \tan 26.7°$$

Now, divide both sides of the last equation by $1 - \cot 53.2° \tan 26.7°$:

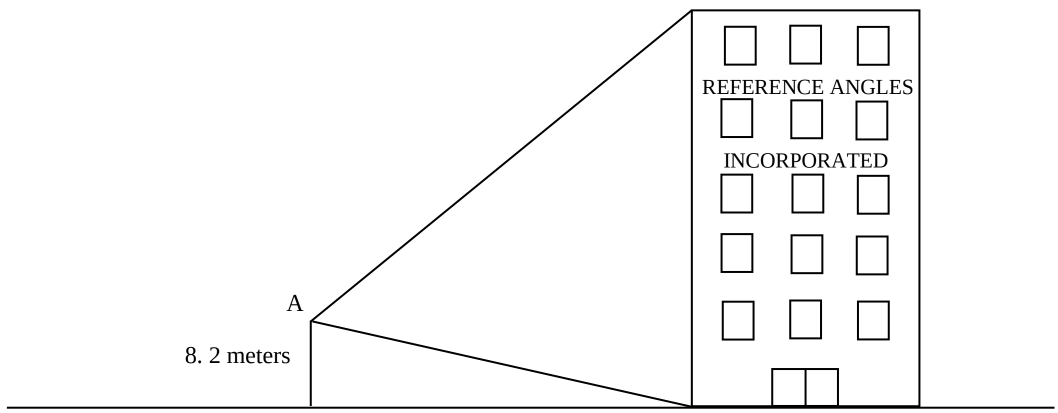
$$y(1 - \cot 53.2° \tan 26.7°) = 25 \tan 26.7° \Rightarrow$$

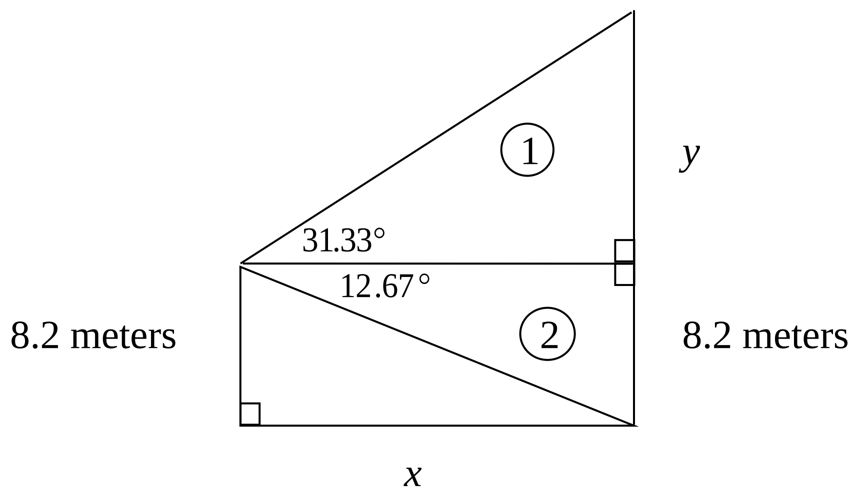
$$y = \frac{25 \tan 26.7°}{1 - \cot 53.2° \tan 26.7°} \approx 20.16$$

NOTE: Here is one way to use your calculator to approximate $\dfrac{25 \tan 26.7°}{1 - \cot 53.2° \tan 26.7°}$ that would work for any scientific calculator:
First, find $\cot 53.2°$: $\cot 53.2° \approx 0.7480955789$. Then multiply this number by $\tan 26.7°$: $\cot 53.2° \tan 26.7° \approx 0.3762528785$. Now, use the $\boxed{+/-}$ key or $\boxed{(-)}$ key to negate this number. Thus, $-\cot 53.2° \tan 26.7° \approx -0.3762528785$. Now, add one to this number. Thus, $1 - \cot 53.2° \tan 26.7° \approx 0.6237471215$. Now, use the $\boxed{x^{-1}}$ key or the $\boxed{1/x}$ key to reciprocate the number. Thus, $\dfrac{1}{1 - \cot 53.2° \tan 26.7°} \approx 1.603213811$. Now, multiply this number by 25 and $\tan 26.7°$.
Of course, there are other ways that you could use your calculator to calculate an approximation for this fraction.

Answer: 20.16 yards

6. From a point A, which is 8.2 meters above the ground, the angle of elevation to the top of a building is $31.33°$ and the angle of depression to the base of the building is $12.67°$. Find the height of the building.





NOTE: The height of the building is $y + 8.2$.

Using Δ 1, we have that $\dfrac{y}{x} = \tan 31.33°$

Using Δ 2, we have that $\dfrac{8.2}{x} = \tan 12.67°$

Use the second equation to solve for *x*:

$$\frac{8.2}{x} = \tan 12.67° \Rightarrow \frac{x}{8.2} = \cot 12.67° \Rightarrow x = 8.2 \cot 12.67°$$

Now, get rid of the fraction in the first equation by multiplying both sides of the equation by *x*:

$$\frac{y}{x} = \tan 31.33^\circ \Rightarrow \quad y = x \tan 31.33^\circ$$

Now, substitute $8.2 \cot 12.67^\circ$ for x in the last equation:

$$y = x \tan 31.33^\circ \Rightarrow \quad y = 8.2 \cot 12.67^\circ \tan 31.33^\circ \approx 22.20$$

Thus, the height $= y + 8.2 = 22.20 + 8.2 = 30.40$

Answer: 30.40 meters

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