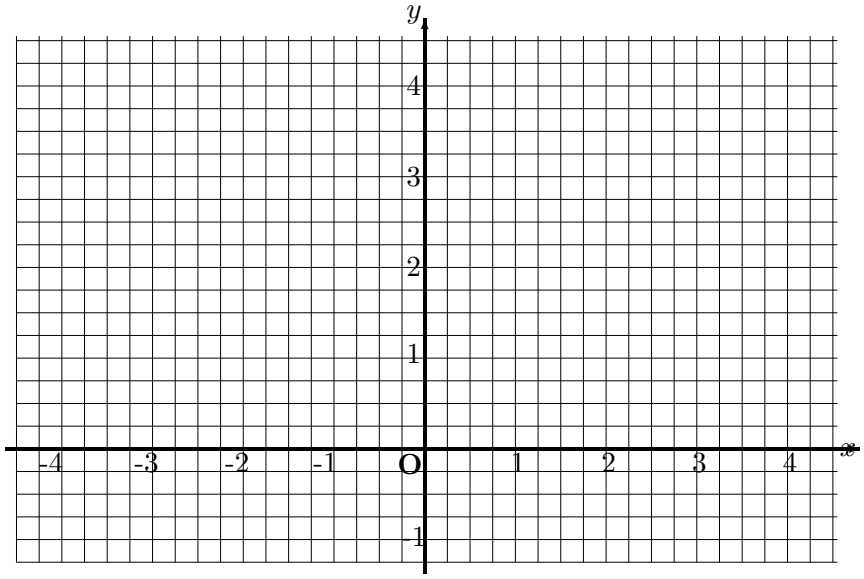
M. Pettet **Math 2850 Test 2, Fall 2009** Name & ID: _____

Write answers in appropriate spaces. Show all work clearly and completely.
Use reverse sides if necessary. Answers should be exact (not rounded off).

1. (8 pt.) Let $f(x,y) = \dfrac{1}{\sqrt{y-x^2}}$

   (a) Describe and sketch the domain of f. (Shade the appropriate region.)

   The domain of f is the set of all points (x,y) such that _____.



2. Explain briefly (in a few *coherent* sentences) why $\lim\limits_{(x,y)\to(0,0)} \dfrac{y^2}{x^2+y^2}$ does not exist.

3. (4 pt.)If $f(x,y,z) = x^2y + 2y^2z$, find an equation for the level surface of the f which passes through the point $(1,2,3)$.

4. (15 pt.) Let $f(x, y) = \sin(x^2 y)$. Find

   (a) $\frac{\partial f}{\partial x}$

   (a)_____

   (b) $\frac{\partial^2 f}{\partial x \partial y}$

   (c)_____

5. (6 pt.) Assume that z is a function of the independent variables x and y such that $xy^2 + yz^3 - 2xyz = 4$. Find an expression for $\frac{\partial z}{\partial y}$ in terms of x, y and z.

   $\frac{\partial z}{\partial y} =$ _____

6. (10 pt.) Let $f(x, y) = x^3 + xy^2$.

   (a) Find the linearization $L(x, y)$ of $f(x, y)$ at $(1, 2)$.

   (a) _____

   (b) Find an equation for the tangent plane to the graph of f at $(1, 2, 5)$.

   (b) _____

   (c) If $z = x^3 + xy^2$, find the differential dz at $(1, 2)$.

   (c) _____

7. (10 pt.) Suppose that $z = ye^x$, where $x = r\cos\theta$ and $y = r\sin\theta$. Using the Chain Rule for functions of more than one variable, write expressions for $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$ in terms of r, θ, x and y.

$\frac{\partial z}{\partial r} =$ _____

$\frac{\partial z}{\partial \theta} =$ _____

8. (16 pt.) Let $f(x, y, z) = 2x^2y + yz^3$.

(a) Find the gradient ∇f at the point $(1, 2, 1)$. (Your answer to other parts of this question may depend on your answer here so check it carefully before continuing.)

(a)_____

(b) What is the directional derivative of f at $(1, 2, 1)$ in the direction of the vector $2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$?

(b)_____

(c) In which direction is the function $f(x, y, z) = 2x^2y + yz^3$ increasing most rapidly at $(1, 2, 1)$?
(Use a unit vector to indicate the correct direction.)

(c)_____

(d) Find an equation for the plane which is tangent to the level surface of f through the point $(1, 2, 1)$.

(d)_____