

Write answers in appropriate spaces. Show all work clearly and completely.

Use reverse sides if necessary. Answers should be exact (*not* rounded off).

1. (8 pt.) Find the center and radius of the sphere whose equation is $x^2 + y^2 + z^2 - 8y + 4z = 0$.

Center: _____

Radius: _____

2. (32 pt.) Let $\mathbf{A} = 3\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$, $\mathbf{B} = 4\mathbf{j} + 3\mathbf{k}$ and $\mathbf{C} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$. Find

(a) $|\mathbf{A}|$

(a) _____

(b) $\mathbf{A} \cdot \mathbf{B}$

(b) _____

(c) the *cosine* of the angle between \mathbf{A} and \mathbf{B} .

(c) _____

(d) the vector projection $proj_{\mathbf{A}}(\mathbf{B})$ of \mathbf{B} on \mathbf{A} .

(d) _____

(e) $\mathbf{A} \times \mathbf{B}$

(e) _____

(f) the area of the triangle two of whose sides are \mathbf{A} and \mathbf{B} .

(f) _____

(g) the volume of the parallelepiped defined by \mathbf{A} , \mathbf{B} and \mathbf{C} .

(g) _____

(h) a unit vector with the same direction as \mathbf{A} .

(h) _____

3. (9 pt.) Find a set of scalar parametric equations for the line through the points $P(4, -1, 3)$ and $Q(3, -2, 5)$.

4. (9 pt.) Find an equation for the plane containing $P(0, 1, 1)$, $Q(1, 0, 3)$ and $R(-2, 1, -1)$.

5. (12 pt.) Sketch the graphs of the following surfaces *in space*. Give a descriptive name to each surface.

(a) $x^2 + y^2 = 4$

(b) $z = 4x^2 + y^2$

(a) Name of surface: _____

(b) Name of surface: _____

6. (15 pt.) For the vector function $\mathbf{r}(t) = (4 \sin t)\mathbf{i} - (4 \cos t)\mathbf{j} + (3t)\mathbf{k}$, find

(a) the unit tangent vector $\mathbf{T}(t)$

(a) _____

(b) the curvature $\kappa(t)$ of the parametric curve defined by $\mathbf{r} = \mathbf{r}(t)$

(b) _____

(c) the length of the curve defined by $\mathbf{r} = \mathbf{r}(t)$ between the points $(0, -4, 0)$ and $(0, 4, 3\pi)$

(d) _____

7. (15 pt.) The position vector of a particle in space at time t is given by $\mathbf{r}(t) = (5 \sin t)\mathbf{i} - (5 \cos t)\mathbf{j} + (12t)\mathbf{k}$ (the same function as in the previous problem). Find

(a) the velocity $\mathbf{v}(t)$

(a) _____

(b) the acceleration $\mathbf{a}(t)$

(b) _____

(c) the speed of the particle (as a function of t)

(c) _____