Introduction to Matlab

- History of Matlab
- Starting Matlab
- Matrix operation
- Useful commands in linear algebra
- Scripts-M file
- Use Matlab to explore the notion of span and the geometry of eigenvalues and eigenvectors.

History of Matlab

- **MATLAB stands for MATrix LABoratory.**
- Cleve Moler is a mathematician and computer programmer. In the mid to late 1970s, he invented MATLAB, a numerical computing package, to give his students at the University of New Mexico easy access to Fortran libraries for numerical computing without writing Fortran. In 1984, he co-founded The MathWorks with Jack Little to commercialize this program.
- MATLAB provides a high-performance technical computing environment. MATLAB combines comprehensive math and graphics functions with a powerful high-level language.

Starting MATLAB

- Press Ctrl-Alt-Delete to begin
- Login on to UTAD with your UTAD Username and Passwordd
- Click "start" ⇒ Programs ⇒ MATLAB ⇒ R2008A
  ⇒ MATLAB R2008 A
- Now you should have MATLAB running on your desktop now.

Set up your working directory

- It's important to set up the right working directory so you can save your works properly.
- First, Change your working directory to your UT H-Drive by the following steps.
- Look for Current Directory. Click on the sign "..." (Browse Folder)

  Move down the cursor to find your UTAD H-Drive (yourutadusername.utoledo.edu ...(H:)

  Now your current are under your UTAD H Drive.
- Now create a MATLAB directory by (right) clicking your mouse on the blank spot of Current Directory. Then select New ⇒ Folder. Then change the name to MATLAB. This is where you save your work. You can create other folders later if you want to organize your files.

Entering and Displaying a Matrix

The MATLAB prompt is $\gg$

To enter a matrix:

- start with [
- separate elements of the matrix with space
- use **;** (semicolon) to mark end of each row
- end the matrix with]

Example 1. Enter the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ into work space.

Solution:

Type the following, followed by Enter (or return key).

$$\gg A = [\ 1\ 2\ 3\ ;\ \ 4\ 5\ 6]$$

You should see

$A =$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Example 2. Enter the vector $u = [\ 1 \quad 2 \quad 3\ ]$ and $v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ into work space.

Type the following, followed by Enter (or return key).

$$\gg u = [\ 1\ 2\ 3]$$

You should see
$u =$

$$[\ 1\ 2\ 3\ ]$$

Type the following, followed by Enter (or return key).

$$\gg v = [\ 1;\ 2;\ 3]$$

You should see

$v =$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Exercise 1. Enter and display the following matrices in MATLAB.
$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$.

Matrix Operation

MATLAB can be used to perform the matrix algebra. The symbols for standard operations are:

Addition: $+$ Substraction: $-$ Multiplication: $*$
Power: $\wedge$ Transpose: $'$

Example 2. Compute AB , A^T, B^2, AC and $D = BC$. What happen if you try DC?

Matrix Operation Solution:
$\gg A * B$
$ans =$

$$\begin{bmatrix} 30 & 36 & 42 \\ 66 & 81 & 96 \end{bmatrix}$$

$\gg A'$
$ans =$

$$\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

$\gg B$^2
$ans =$

$$\begin{bmatrix} 30 & 36 & 42 \\ 66 & 81 & 96 \\ 102 & 126 & 150 \end{bmatrix}$$

$\gg A * C$
$ans =$

$$\begin{bmatrix} 22 & 28 \\ 49 & 64 \end{bmatrix}$$

$\gg D = B * C$
$D =$

$$\begin{bmatrix} 22 & 28 \\ 49 & 64 \\ 76 & 100 \end{bmatrix}$$

$\gg D * C$
??? Error using $\Longrightarrow$ mtimes
Inner matrix dimensions must agree.

Since DC doesn't make sense, You will see the error information.

Now the matrices C and D have the same dimension. We can do the following example. Compute $C + D$ and $2C - 3D$.

Solution:

$\gg C + D$
$ans =$

$$\begin{bmatrix} 23 & 30 \\ 52 & 68 \\ 81 & 106 \end{bmatrix}$$

$\gg 2 * C - 3 * D$
$ans =$

$$\begin{bmatrix} -64 & -80 \\ -141 & -184 \\ -218 & -288 \end{bmatrix}$$

Don't forget $*$ sign when you multiply a number with a matrix.

Submatrices, Rows and Columns of a Matrix and other commands

$A(i,j)$: returns the ij entry of the matrix A

$A(i,:)$: returns the $i-$th row of A

$A(:,j)$: returns the $j-$th column of A.

$A(p:q, r:s)$: returns the submatrix from row p to row q and column r to column s. (Here $p \leq q$ and $r \leq s$.)

eye(n): returns a $n \times n$ identity matrix.

zeros(m,n): returns a $m \times n$ zero matrix.

rand(m,n): returns a $m \times n$ random matrix.

Linear algebra commands

rref(A): returns the row reduced echelon form of A.

$det(A)$: returns the determinant of the square matrix A.

trace(A): returns the sum of the diagonal elements of the matrix A

size(A): returns the size of a matrix. $x = linsolve(A,b)$ solves the linear system $Ax = b$ using LU factorization.

$rank(A)$: returns the rank of A

$orth(A)$: returns an orthonormal basis for the range (column space) of A.

$null(A)$: returns the orthonormal basis for the null space of of A.

$Z = null(A,'r')$: returns a "rational" basis for the null space obtained from the reduced row echelon form. $A \cdot Z$ is zero.

Linear algebra commands

dot(u,v): returns a scalar product of the vectors u and v. u and v must be vectors of the same length.

cross(u,v) returns the cross product of the vectors u and v. u and v must be 3 element vectors.

svd(A): returns a vector containing the singular values.

$[U,S,V] = \text{svd}(A)$: returns a diagonal matrix S, of the same dimension as A and with nonnegative diagonal elements in decreasing order, and unitary matrices U and V so that $A = USV^T$.

$[L,U] = lu(A)$ stores an upper triangular matrix in U and a "psychologically lower triangular matrix" (i.e. a product of lower triangular and permutation matrices) in L, so that $A = LU$. A can be rectangular.

$[L,U,P] = lu(A)$ returns unit lower triangular matrix L, upper triangular matrix U, and permutation matrix P so that $PA = LU$.

Linear algebra commands

eig(A): returns a vector containing the eigenvalues of a square matrix A

$[V, D]=$ eig(A): returns a diagonal matrix D of eigenvalues and a full matrix V whose columns are the corresponding eigenvectors so that $AV = VD$

svd(A): returns a vector containing the singular values. $[U, S, V] =$ svd(A): returns a diagonal matrix S, of the same dimension as A and with nonnegative diagonal elements in decreasing order, and unitary matrices U and V so that $A = USV^T$.

$[L, U] = lu(A)$ stores an upper triangular matrix in U and a "psychologically lower triangular matrix" (i.e. a product of lower triangular and permutation matrices) in L, so that $A = LU$. A can be rectangular.

$[L, U, P] = lu(A)$ returns unit lower triangular matrix L, upper triangular matrix U, and permutation matrix P so that $PA = LU$.

How to get help

- Type "help command" if you are interested in the detail of the command.
- For example, if you type "help svd".
  You will see the detail about the command "svd".
- You can also scroll down the "help" window and click on "Matlab help". Then click on "Mathematics". Then click on "Matrices and Linear Algebra". Then you can choose the topics that you are interested in.

Scripts M-files

- A script M-file is just a sequence of MATLAB commands stored in a text file that has ".m" as its extension. You can use the m editor on MATLAB to create a m file or any other text editor (like notepad but not WORD). After creating a m-file, you just need to type the name of the file (without .m) to run the command in the m-file.
- You can download fern.m, eigshow.m and spanshow.m from $http : //math.utoledo.edu/ \sim mtsui/techseminar/fern.m$ $http : //math.utoledo.edu/ \sim mtsui/techseminar/eigshow.m$ and $http : //math.utoledo.edu/ \sim mtsui/techseminar/spa$
  Make sure that you save these files as fern.m, eigshow.m and spanshow.m. Save these three files in your current directory.
- Now type fern.
  You should see the fractal fern.

Scripts M-files

- Example. Create a m-file named project1.m that creates a matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ and compute $B = A^3 - 6A^2 + 5A - 3I$.

  Solution: First Click on file. Then click on "New" and choose "M-file".

  Type the following:
  $A = [\ 1\ 2\ 1\ ; 1\ 2\ 2\ ; 2\ 1\ 3]$
  $A\hat{}3 - 6 * A\hat{}2 + 5 * A + 3 * eye(3)$
  Then save the file as project1.m to your working directory. Note that each command is in different lines.

  Now type project1. You should see the results. $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

eigshow

- Example 1. Type eigshow to explore the geometry of the eigenvalues and eigenvectors of a 2×2 matrix.

- Example 2. Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$. Type eigshow(A) the geometry of the eigenvalues and eigenvectors of a 2×2 matrix.

spanshow

- A is a 3×3 matrix. $spanshow(A)$: Show the plane spanned by the first two column vectors and the the line span by the third column vector.

- Example 1. Create $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ and type spanshow(A).

- Example 2. Create $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix}$ and type spanshow(A).