

1.1 Angles and Their Measurement

August 27, 2012

Definition 1 *Two angles are adjacent if they lie in the same plane, they share a common side, and the interiors of the angles have no point in common.*

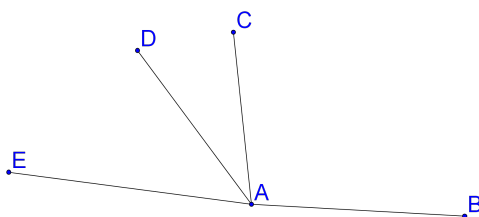


Figure 1:

Example 1 (a) *What is the angle that $\angle BAC$ is adjacent to?*
(b) *Is $\angle BAD$ adjacent to $\angle CAE$?*

Answer: $\angle BAC$ is adjacent to $\angle CAD$.
 $\angle BAD$ is not adjacent to $\angle CAE$ because the interiors of these two angles overlap.

Definition 2 *The common side of two adjacent angles of equal measure (angle) is called an angle bisector.*

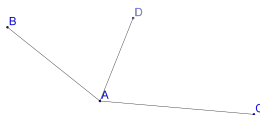


Figure 2:

Example 2 *What is the angle bisector of the above figure?*

Definition 3 *Two angles form a linear pair if they are adjacent and the non common sides are opposite rays.*

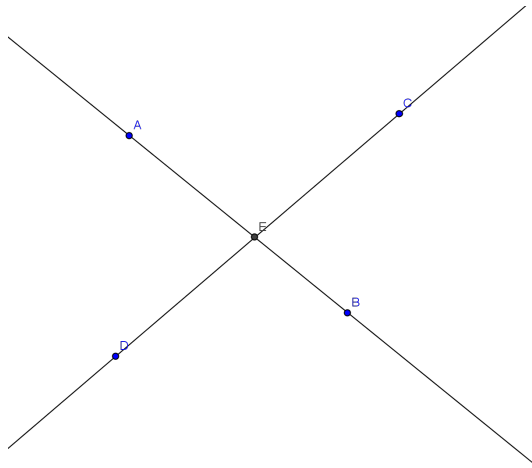


Figure 3:

Definition 4 Two angles are vertical if their sides form two pair of opposite rays.

Example 3 (a) $\angle CEA$ and \angle _____ form a linear pair.

(b) $\angle CEA$ and \angle _____ are vertical .

Definition 5 Two angles are supplementary if the sum of their measures is 180° (or π); each angle is called the supplement of the other.

Remark Two angles that form a linear pair are supplementary. But two angles that are supplementary may not form a linear pair.

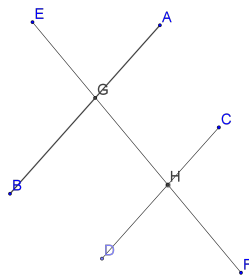


Figure 4:

Example 4 (a) Find all the angles in the above figure that are supplementary to $\angle AGE$. Note that \overline{AB} and \overline{CD} are parallel.

(b) Find all the angles in the above figure that are supplementary to $\angle AGE$ and don't form a linear pair with $\angle AGE$.

Definition 6 Two angles are complementary if the sum of their measures is 90° (or $\frac{\pi}{2}$); each angle is called the complement of the other.

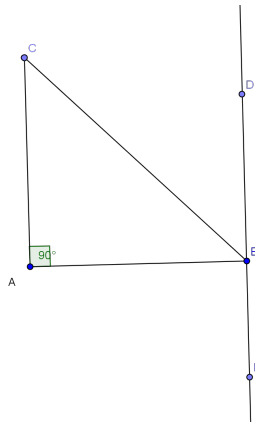


Figure 5:

Example 5 (a) Find all the angles in the above figure that are complementary to $\angle CBA$.

Note that \overline{CA} and \overline{DE} are parallel.

Definition 7 An angle measuring less than 90^0 is called acute.
 An angle measuring more than 90^0 is called obtuse.
 An 90^0 angle is called a right angle.
 An 180^0 angle is called a straight angle.

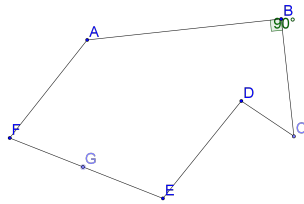


Figure 6:

Example 6 $\angle FAB$, $\angle CDE$ and $\angle DEG$ are obtuse.
 $\angle BCD$ and $\angle GFA$ are acute. $\angle ABC$ is a right angle. $\angle EGF$ is a straight angle.

Definition 8 Two angles are congruent if they have the same measure.
 Two line segments are congruent if they have the same length.

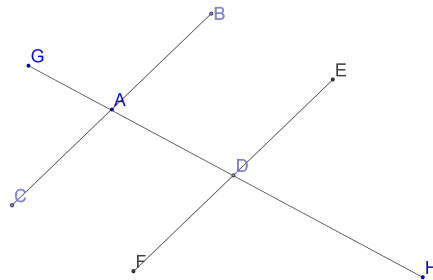


Figure 7:

Example 7 In the figure above, \overline{BE} is parallel to \overline{AD} and \overline{BC} is parallel to \overline{EF} . $\angle BAG$, $\angle CAD$, $\angle EDA$ and $\angle FDH$ are congruent in the above picture.
 \overline{AB} and \overline{DE} are congruent. \overline{GA} and \overline{DH} are not congruent.

Lecture note on August 29, 2012

Remark 1 We denote $\angle\alpha \cong \angle\beta$ if $\angle\alpha$ is congruent to $\angle\beta$.

After introducing preceding definitions, we can derive the following theorems.

Theorem 0.1 1. *Supplementary of congruent angles are congruent.*
2. *complementary of congruent angles are congruent.*
3. *The sum of the measure of two angles in a linear pair is 180^0 (π).*

Theorem 0.2 *Vertical angles are congruent.*

Example 8 *When two lines intersect, they form four angles if they don't overlap. If one of the angles is a right angle, then all four angles are right angles. We say these two lines are perpendicular and is denoted by the symbol \perp*