Lecture note on August 29, 2012

Remark 1 *We denote $\angle A \cong \angle B$ if $\angle A$ is congruent to $\angle B$.*
We denote $\overline{AB} \cong \overline{CD}$ if $\overline{AB}$ is congruent to $\overline{CD}$ (i.e they have the same length).
We will denote the measure of an angle A by $m(\angle A)$

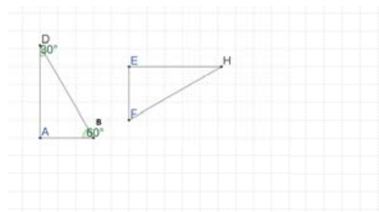

Figure 8:

Example 8 *1. $m(\angle A) =?$ $m(\angle B) =?$ $m(\angle D) =?$.*
2. $\overline{BD} \cong ?$ $\angle D \cong ?$

After introducing preceding definitions, we can state the following theorems.

Theorem 0.1 *1. Supplementary of congruent angles are congruent.*
2. Complementary of congruent angles are congruent.
3. The sum of the measure of two angles in a linear pair is 180^0 (π).

We will prove the first statement. The proofs of the other two results are similar.
Proof. Let A, B, C, D be four angles such that $\angle A \cong \angle C$, $\angle B$ is supplementary to $\angle A$ and $\angle D$ is supplementary to $\angle C$. By definition, we have $m(\angle A) = m(\angle C)$, $m(\angle B) + m(\angle A) = 180°$ and $m(\angle C) + m(\angle D) = 180°$. Thus we have $m(\angle B) + m(\angle A) = m(\angle C) + m(\angle D)$. Using $m(\angle A) = m(\angle C)$, we get $m(\angle B) = m(\angle D)$. Thus $\angle B \cong \angle D$. □

Theorem 0.2 *Vertical angles are congruent.*

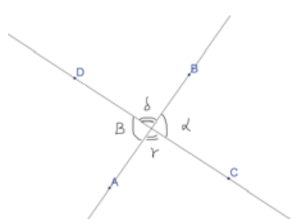

Figure 9:

Proof. In the above figure, we want to show that $\alpha = \beta$ and $\gamma = \delta$. Note that α and δ form a linear pair and β and δ form a linear pair. We have $\alpha + \delta = 180°$. $\beta + \delta = 180°$. This gives $\alpha + \delta = \beta + \delta$ and implies that $\alpha = \beta$. Similarly, we can show that $\gamma = \delta$. □

Definition 9 *Two lines l and m are perpendicular if there exists a point A that lies on both l and m and there exists points $B \in l$ and $C \in m$ such that $\angle BAC$ is a right angle. We denote it by $l \perp m$ if l and m are perpendicular. We also denote it by $\urcorner$ in the figure.*

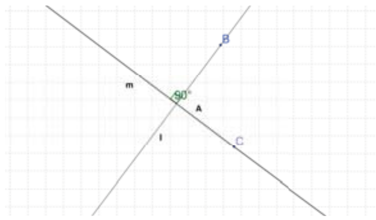

Figure 10:

Definition 10 *A perpendicular bisector of a segment is a line intersect a segment at its midpoint and is perpendicular to the segment.*

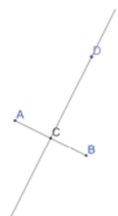

Figure 11:

Example 9 *In the above figure, which segment is the perpendicular bisector of $\overline{AB}$?*

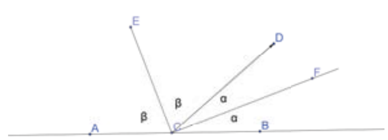

Figure 12:

Example 10 *In the above figure, $\angle BCD$ and $\angle DCA$ form a linear pair. The rays $\overrightarrow{CF}$ and $\overrightarrow{CE}$ are their angle bisectors, respectively. Prove that $\overrightarrow{CF} \perp \overrightarrow{CE}$.*

Proof. Since $\angle BCD$ and $\angle DCA$ form a linear pair, we have $m(\angle BCD) + m(\angle DCA) = 180°$. Because rays $\overrightarrow{CF}$ and $\overrightarrow{CE}$ are angle bisectors, we get $m(\angle BCD) = 2\alpha$ and $m(\angle DCA) = 2\beta$. This implies that $2\alpha + 2\beta = 180°$ and $\alpha + \beta = 90°$. Thus $m(\angle FCE) = 90°$ and $\overrightarrow{CF} \perp \overrightarrow{CE}$. $\square$