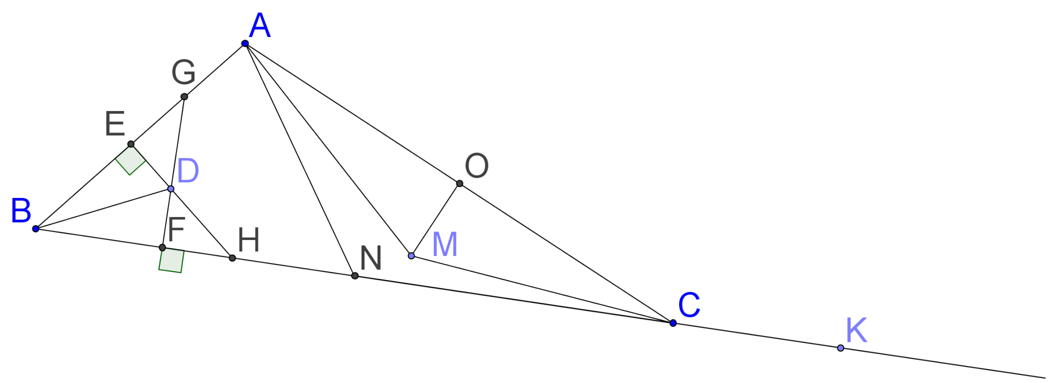
Review Problems for Midterm I

Midterm I: 2pm - 2:50 pm Friday, September 21 at UH1000 Newton Lab

Topics: 1.1 and 1.2

Office Hours before the midterm: Thursday 4:30-5:30 pm, F 10-11 am at UH2080B.



1.

Given a figure above and a triangle $\triangle ABC$ which have the following properties.

(i) $\overline{DE} \cong \overline{DF}$

(ii) $\overline{DE} \perp \overline{AB}$ and $\overline{DF} \perp \overline{BC}$

(iii) $\overline{MA} \cong \overline{MC}$

(iv) O is the midpoint of $\overline{AC}$.

(v) N is the midpoint of $\overline{BC}$.

(vi) B, C and K lie on the line.

(vii) There is exactly one median, angle bisector and perpendicular bisector in the above figure.

Answer the following questions.

(a) What is the exterior angle of $\angle BCA$?
Solution: $\angle ACK$

(b) What is the angle that make a linear pair with $\angle EDG$?
Solution: $\angle GDH$ or $\angle EDF$

(c) What is the angle vertical to $\angle EDG$?
Solution: $\angle FDH$

(d) Which triangle is $\triangle BDE$ congruent to? Justify your answer.
Solution: $\triangle BDE \cong \triangle BDF$.
Note that $\triangle BDE$ and $\triangle BDF$ are right triangles. Because $\overline{DE} \cong \overline{DF}$ and $\overline{BD} \cong \overline{BD}$. We can use the hypotenuse-leg condition for right triangle to conclude that $\triangle BDE \cong \triangle BDF$.

(e) Which segment is an angle bisector? Justify your answer.
Solution: $\overline{BD}$ is the angle bisector of $\angle EBH$. From previous question, we know that $\triangle BDE \cong \triangle BDF$ which implies $\angle EBD \cong \angle FBD$. Thus $\overline{BD}$ is the angle bisector of $\angle EBH$.

(f) Which segment is an perpendicular bisector? Justify your answer.
Solution: $\overline{MQ}$ is the perpendicular bisector of $\overline{CA}$. Since $\overline{OA} \cong \overline{OC}$, $\overline{AM} \cong \overline{CM}$ and $\overline{MO} \cong \overline{MO}$, we have $\triangle MOA \cong \triangle MOC$ by SSS. So $\angle MOA \cong \angle MOC$. Also $m(\angle MOA) + m(\angle MOC) = 180^0$. Then $m(\angle MOA) = m(\angle MOC) = 90^0$. Thus $\overline{MQ}$ is the perpendicular bisector of $\overline{CA}$.

(g) Which segment is an median of $\triangle ABC$? Why?
Solution: $\overline{AN}$ is the median of the triangle $\triangle ABC$ because N is the midpoint of $\overline{BC}$.

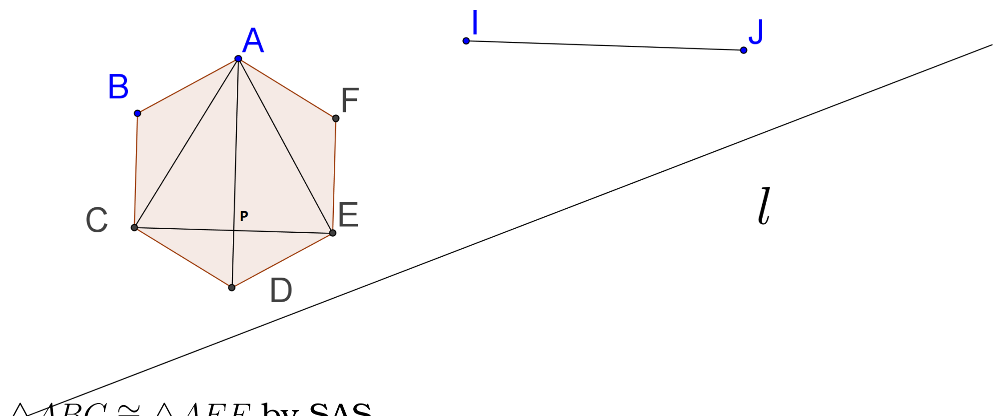
(h) Which triangle is $\triangle EDG$ congruent to? Justify your answer.
Solution: $\triangle EDG \cong \triangle FDH$. Note that $\triangle EDG$ and $\triangle FDH$ are right triangles. Since $\angle GED \cong \angle HFD$ (right angle), $\overline{DE} \cong \overline{DF}$ and $\angle EDG \cong \angle FDH$, we have $\triangle EDG \cong \triangle FDH$ by ASA.

(i) Which of the following angle is bigger? $\angle ACK$ or $\angle BAC$. Why?
Solution: $m(\angle ACK) > m(\angle BAC)$ because $\angle ACK$ is the exterior angle of $\angle BAC$ and $\angle BAC$ is the remote angle of $\angle BAC$. .

(j) Which of the following angle is bigger? $\angle ACK$ or $\angle EBD$. Why?
Solution: $m(\angle ACK) > m(\angle EBD)$. Because $\angle ACK$ is the exterior angle of $\angle BAC$, $\angle ABC$ is the remote angle of $\angle BAC$ and $\overline{BD}$ is the angle bisector of $\angle ABC$. We have $m(\angle ACK) > m(\angle ABC) = 2m(\angle EBD) > m(\angle EBD)$.

Given a hexagon above which satisfies the following properties.
(i) $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DE} \cong \overline{EF} \cong \overline{FA}$
(ii) $\angle A \cong \angle B \cong \angle C \cong \angle D \cong \angle E \cong \angle F$
Answer the following questions.

(a) Prove that $\triangle ABC \cong \triangle AEF$ (there is a typo in the original problem).
Solution: Since $\overline{AB} \cong \overline{AF}$, $\angle ABC \cong \angle AFE$ and $\overline{BC} \cong \overline{FE}$, we have

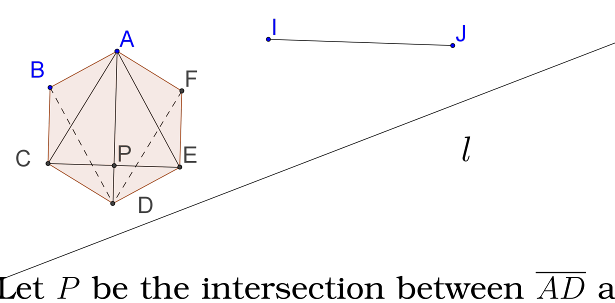


2. $\triangle ABC \cong \triangle AEF$ by SAS.

(b) Prove that $\triangle ACE$ is a equilateral triangle.
Solution: From previous question, we know that $\triangle ABC \cong \triangle AEF$ which implies $\overline{AC} \cong \overline{AE}$. Similarly, we can also prove that $\triangle ABC \cong \triangle CDE$ which implies $\overline{AC} \cong \overline{CE}$. Thus we have $\overline{AC} \cong \overline{AE} \cong \overline{CE}$ and $\triangle ACE$ is a equilateral triangle.

(c) Which triangle is $\triangle ACD$ congruent to? Justify your answer. Solution: $\triangle ACD \cong \triangle AED$. Since $\overline{AC} \cong \overline{AE}$ (from a), $\overline{CD} \cong \overline{ED}$ and $\overline{AD} \cong \overline{AD}$, we have $\triangle ACD \cong \triangle ADE$ by SSS.

(d) Prove that $\overline{AD} \perp \overline{CE}$.
Solution: Method 1: Since $\overline{AC} \cong \overline{AE}$ (from a) and $\overline{DC} \cong \overline{DE}$, so the quadrilateral $ACDE$ is a kite. Thus the diagonal $\overline{AD} \perp \overline{CE}$.



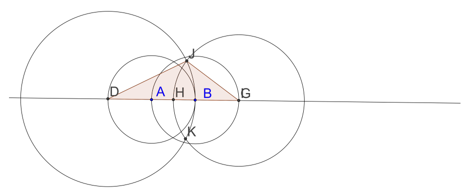
Method 2: Let P be the intersection between $\overline{AD}$ and $\overline{CE}$. Since $\overline{AC} \cong \overline{AE}$, $\angle CAP \cong \angle EAP$ (from c) and $\overline{AP} \cong \overline{AP}$, we have $\triangle ACP \cong \triangle AEP$ by SAS. So $\angle CPA \cong \angle EPA$. Note that $\angle CPA$ and $\angle EPA$ form a linear pair. So $\angle CPA$ is a right angle and $\overline{AD} \perp \overline{CE}$.

(e) Why is $\overline{AD}$ the angle bisector of $\angle BAF$?
Solution: Look at $\triangle ABD$ and $\triangle AFD$. Since $\overline{AB} \cong \overline{AF}$, $\overline{BD} \cong \overline{FD}$ (b/c $\triangle BCD \cong \triangle FED$) and $\overline{AD} \cong \overline{AD}$, we have $\triangle ABD \cong \triangle AFD$ by SSS. Then $\angle BAD \cong \angle FAD$ and $\overline{AD}$ the angle bisector of $\angle BAF$.

3. Which of the following constructions can be achieved by Euclidean construction? Please justify your answer and use GeoGebra to construct it if it is an Euclidean construction.

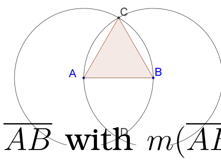
(a) Construct a triangle whose sides are $2\ inch$, $1.5\ inch$ and $1\ inch$.
Solution: This is not an Euclidean construction. Because Euclidean construction can not involve specific length.

(b) Given a segment $\overline{AB}$ with $m(\overline{AB}) > 0$. Construct a triangle whose sides are $3m(\overline{AB})$, $1.5m(\overline{AB})$ and $2m(\overline{AB})$ (there is a typo in the original problem. It should be $2m(\overline{AB})$).



Solution: This is an Euclidean construction. Note that finding a midpoint is an Euclidean construction as explained in class.
We start with a line $\overleftrightarrow{AB}$. 1. Construct a circle whose center is A with radius $m(\overline{AB})$. This circle intersects with the line $\overleftrightarrow{AB}$ at D and B. 2. Construct a circle whose center is B with radius $m(\overline{AB})$. This circle intersects with the line $\overleftrightarrow{AB}$ at G and A. 3. Now $m(\overline{DG}) = 3m(\overline{AB})$. Let H be the midpoint of D and G. Then $m(\overline{HG}) = 1.5m(\overline{AB})$. 4. Now construct a circle whose center is D with radius $m(\overline{DB}) = 2m(\overline{AB})$ and construct a circle whose center is G with radius $m(\overline{GH}) = 1.5m(\overline{AB})$. These two circles intersect at two points J and K.
Then $\triangle JDG$ is a triangle with $m(\overline{DG}) = 3m(\overline{AB})$, $m(\overline{GJ}) = 1.5m(\overline{AB})$ and $m(\overline{JD}) = 2m(\overline{AB})$.

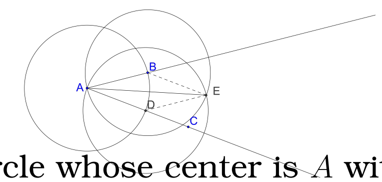
(c) Construct an equilateral triangle whose sides are all $1\ inch$ long.
Solution: This is not an Euclidean construction. Because Euclidean construction can not involve specific length.



(d) Given a segment $\overline{AB}$ with $m(\overline{AB}) > 0$. Construct an equilateral triangle $\triangle ABC$
Solution: Start with a segment $\overline{AB}$. 1. Construct a circle whose center is A with radius $m(\overline{AB})$ and another circle whose center is B with radius $m(\overline{AB})$. 2. These two circles intersect at C and D. Then $\triangle ABC$ is an equilateral triangle.

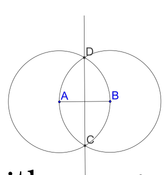
(e) Construct a triangles whose interior angles are $25°$, $85°$ and $70°$.
Solution: This is not an Euclidean construction. Because Euclidean construction can not be used construct this specific angle. (Euclidean construction can be to used construct some special triangle like $30^0 - 60^0 - 90^0$ or $45^0 - 45^0 - 90^0$ triangles.)

(f) The angle bisector of a nonzero angle.
Solution: Start with a angle $\angle A$ whose sides are $\overrightarrow{AB}$ and $\overrightarrow{AC}$.



Construct a circle whose center is A with radius $m(\overline{AB})$. 2. This circle intersect the line $\overrightarrow{AB}$ at B and intersect the line $\overrightarrow{AC}$ at D. 3. Construct a circle whose center is B with radius $m(\overline{AB})$ and another circle whose center is D with radius $m(\overline{AD})$. These two circles at A and E. Then $\overline{AE}$ is the angle bisector of $\angle A$.

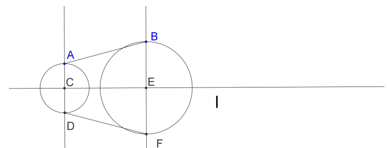
(g) The perpendicular bisector of a segment $\overline{AB}$.



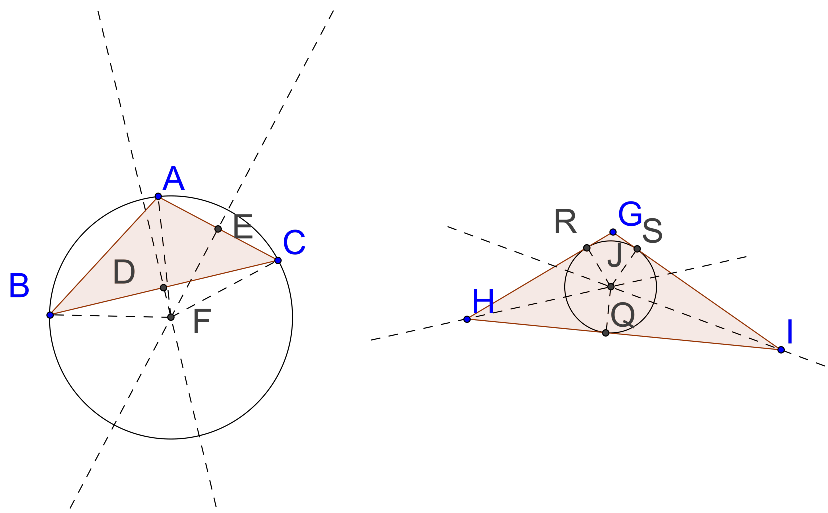
Solution:Start with a segment $\overline{AB}$. 1. Construct a circle whose center is A with radius $m(\overline{AB})$ and another circle whose center is B with radius $m(\overline{AB})$. 2. These two circles intersect at C and D. Then $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$.

(h) Given a line l and a segment $\overline{AB}$ where $\overline{AB}$ doesn't intersect with the line l. See the picture. Construct the reflection of $\overline{AB}$ with respect to l.

Solution: Recall that constructing a line through a point that



is perpendicular to a line is an Euclidean construction. 1. Construct a line through A that is perpendicular to the line l. This perpendicular line intersects the line l at C. Now construct a circle whose center is at C and radius is $\overline{CA}$. This circle with intersect the perpendicular line $\overleftrightarrow{AC}$ at C and D. 2. Construct a line through B that is perpendicular to the line l. This perpendicular line intersects the line l at E. Now construct a circle whose center is at E and radius is $\overline{EB}$. This circle with intersect the perpendicular line $\overleftrightarrow{EB}$ at B and F. 3. The segment $\overline{DF}$ is the reflection of the segment $\overline{AB}$ with respect to l.



4.

Given a triangles $\triangle ABC$ in the above figure which satisfies the following properties.
(i) $\overline{EF}$ is the perpendicular bisector of $\overline{AC}$.
(ii) $\overline{FD}$ is the perpendicular bisector of $\overline{BC}$.
Prove that $\overline{FA} \cong \overline{FB} \cong \overline{FC}$.
Solution: Let us first look at the triangle $\triangle FAC$. Because $\overline{FE}$ is the perpendicular bisector of $\overline{AC}$. We have $\overline{FA} \cong \overline{FC}$ ($\triangle FCE \cong FAE$ by SAS b/c $\overline{EA} \cong \overline{EC}$, $\angle FEA \cong \angle FEC(right\ angle)$ and $\overline{FE} \cong \overline{FE}$.).

Now let us look at the triangle $\triangle FBC$. Because $\overline{FD}$ is the perpendicular bisector of $\overline{BC}$. We have $\overline{FB} \cong \overline{FC}$ (the proof is similar to the proof $\overline{FA} \cong \overline{FC}$). Thus we have $\overline{FA} \cong \overline{FB} \cong \overline{FC}$.

5. Given a triangles $\triangle GHI$ in the above figure which satisfies the following properties.
(i) $\overline{HJ}$ is the angle bisector of $\angle H$.
(ii) $\overline{IJ}$ is the angle bisector of $\angle I$.
(iii) $\overline{JQ} \perp \overline{HI}$, $\overline{JR} \perp \overline{HG}$ and $\overline{JS} \perp \overline{GI}$.

Prove that $\overline{JR} \cong \overline{JQ} \cong \overline{JS}$.
Solution: Let us first look at the triangle $\triangle JRH$ and $\triangle JQH$. We have $\triangle JRH \cong JQH$ by acute angle-hypotenuse condition for right triangle b/c $\overline{HJ}$ is the angle bisector of $\angle H$. So $\angle RHJ \cong \angle QJH$. Also we have $\overline{HJ} \cong \overline{HJ}$. Thus we get $\overline{JR} \cong \overline{JQ}$.

Now let us look at the triangle $\triangle JQI$ and $\triangle JSI$. We have $\triangle JQI \cong \triangle JSI$ by similar argument. This implies $\overline{JS} \cong \overline{JQ}$. Therefore $\overline{JR} \cong \overline{JQ} \cong \overline{JS}$.