HW 5 Solution

1

$$\text{(7.1 Problem 20)} \int (x^2-2x)(x^3-3x^2+3)^{\frac{2}{3}}dx$$

Solution: Let $u = x^3-3x^2+3$. Then $du = (3x^2-6x)dx$ and $\frac{du}{3} = (x^2-2x)dx$. So $\int(x^2-2x)(x^3-3x^2+3)^{\frac{2}{3}}dx = \int u^{\frac{2}{3}}\frac{du}{3} = \frac{1}{3}\cdot\frac{3}{5}u^{\frac{5}{3}}+C = \frac{1}{5}u^{\frac{5}{3}}+C = \frac{1}{5}(x^3-3x^2+3)^{\frac{5}{3}}+C$.

2

$$\text{(7.1 Problem 22)} \int \frac{x^2-1}{x^3-3x+1}dx$$

Solution: Let $u = x^3-3x+1$. Then $du = (3x^2-3)dx$ and $\frac{du}{3} = (x^2-1)dx$. So $\int \frac{x^2-1}{x^3-3x+1}dx = \int \frac{1}{x^3-3x+1}\cdot(x^2-1)dx = \int \frac{1}{u}\cdot\frac{du}{3} = \frac{1}{3}\int\frac{1}{u}du = \frac{1}{3}\ln|u|+C = \frac{1}{3}\ln|x^3-3x+1|+C$.

3

$$\text{(7.1 Problem 28)} \int \sec^2(x)e^{\tan(x)}dx$$

Solution: Let $u = \tan(x)$. Then $du = \sec^2(x)dx$. So $\int\sec^2(x)e^{\tan(x)}dx = \int e^{\tan(x)}\sec^2(x)dx = \int e^u du = e^u + C = e^{\tan(x)}+C$.

4

$$\text{(7.1 Problem 36)} \int \sqrt{1+\ln x}\,\frac{\ln x}{x}dx$$

Solution: Let $u = 1+\ln x$. Then $du = \frac{1}{x}dx$ and $\ln x = u-1$ So $\int\sqrt{1+\ln x}\,\frac{\ln x}{x}dx = \int\sqrt{1+\ln x}\cdot\ln x\cdot\frac{1}{x}dx = \int\sqrt{u}\cdot(u-1)du = \int u^{\frac{1}{2}}\cdot(u-1)du = \int(u^{\frac{3}{2}}-u^{\frac{1}{2}})du = \frac{2}{5}u^{\frac{5}{2}}-\frac{2}{3}u^{\frac{3}{2}}+C = \frac{2}{5}(1+\ln x)^{\frac{5}{2}}-\frac{2}{3}(1+\ln x)^{\frac{3}{2}}+C$.

5

$$\text{(7.1 Problem 44)} \int_1^2 x^5\sqrt{x^3+2}dx$$

Solution: Let $u = x^3+2$. Then $du = 3x^2dx$, $\frac{du}{3} = x^2dx$ and $x^3 = u-2$. Also $x=1$ gives $u = 1^3+2 = 3$ and $x=2$ gives $u = 2^3+2 = 8+2 = 10$. So $\int_1^2 x^5\sqrt{x^3+2}dx = \int_1^2 x^3\sqrt{x^3+2}x^2dx = \int_3^{10}(u-2)\sqrt{u}\frac{du}{3} = \frac{1}{3}\int_3^{10}(u-2)u^{\frac{1}{2}}du = \frac{1}{3}\int_3^{10}(u^{\frac{3}{2}}-2u^{\frac{1}{2}})du = \frac{1}{3}(\frac{2}{5}u^{\frac{5}{2}}-2\cdot\frac{2}{3}u^{\frac{3}{2}})\Big|_3^{10} = (\frac{2}{15}u^{\frac{5}{2}}-\frac{4}{9}u^{\frac{3}{2}})\Big|_3^{10} = (\frac{2}{15}\cdot 10^{\frac{5}{2}}-\frac{4}{9}\cdot 10^{\frac{3}{2}}) - \left(\frac{2}{15}\cdot 3^{\frac{5}{2}}-\frac{4}{9}\cdot 3^{\frac{3}{2}}\right) = \frac{2}{15}\cdot 10^{\frac{5}{2}}-\frac{4}{9}\cdot 10^{\frac{3}{2}}-\frac{2}{15}\cdot 3^{\frac{5}{2}}+\frac{4}{9}\cdot 3^{\frac{3}{2}}$.

6

$$\text{(7.1 Problem 52)} \int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos^2(x)}dx$$

Solution: Let $u = \cos(x)$. Then $du = -\sin(x)dx$ and $\sin(x)dx = -du$. Also $x = 0$ gives $u = \cos(0) = 1$ and $x = \frac{\pi}{3}$ gives $u = \cos(\frac{\pi}{3}) = \frac{1}{2}$. So

$\int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos^2(x)} dx = \int_0^{\frac{\pi}{3}} \frac{1}{\cos^2(x)} \cdot \sin(x) dx = \int_1^{\frac{1}{2}} \frac{1}{u^2}(-1) du = -\int_1^{\frac{1}{2}} \frac{1}{u^2} du = \frac{1}{u}\Big|_1^{\frac{1}{2}} = \frac{1}{\frac{1}{2}} - \frac{1}{1} = 2 - 1 = 1.$

7

$$\text{(7.2 Problem 10)} \int 2x^2 e^{-x} dx$$

Solution: Let $u = 2x^2$ and $dv = e^{-x}dx$. Then $du = 4xdx$ and $v = \int e^{-x}dx = -e^{-x}$. So $\int 2x^2e^{-x}dx = 2x^2(-e^{-x}) - \int(-e^{-x}) \cdot 4xdx = -2x^2e^{-x} + 4\int xe^{-x}dx$.

Next we find $\int xe^{-x}dx$ using integration by parts again. Let $u = x$ and $dv = e^{-x}dx$. Then $du = dx$ and $v = \int e^{-x}dx = -e^{-x}$. So $\int xe^{-x}dx = x(-e^{-x}) - \int(-e^{-x})dx = -xe^{-x} + \int e^{-x}dx = -xe^{-x} - e^{-x} + C$.

Combining $\int 2x^2e^{-x}dx = -2x^2e^{-x} + 4\int xe^{-x}dx$ and $\int xe^{-x}dx = -xe^{-x} - e^{-x} + C$, we have $\int 2x^2e^{-x}dx = -2x^2e^{-x} - 4xe^{-x} - 4e^{-x} + C$.

8

$$\text{(7.2 Problem 12)} \int x^2 \ln x dx$$

Solution: Let $u = \ln(x)$ and $dv = x^2dx$. Then $du = \frac{1}{x}dx$ and $v = \int x^2dx = \frac{x^3}{3}$. So $\int x^2 \ln x dx = \int \ln(x) \cdot x^2 dx = \ln(x) \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x}dx = \frac{x^3\ln(x)}{3} - \int \frac{x^2}{3}dx = \frac{x^3\ln(x)}{3} - \frac{x^3}{9} + C$.

9

$$\text{(7.2 Problem 18)} \int_0^{\frac{\pi}{4}} 2x\cos(x)dx$$

Let $u = 2x$ and $dv = \cos(x)dx$. Then $du = 2dx$ and $v = \int \cos(x)dx = \sin(x)$. So $\int 2x\cos(x)dxdx = 2x\sin(x) - \int \sin(x) \cdot 2dx = 2x\sin(x) - 2\int \sin(x)dx = 2x\sin(x) + 2\cos(x) + C$.

Thus $\int_0^{\frac{\pi}{4}} 2x\cos(x)dx = 2x\sin(x) + 2\cos(x)\Big|_0^{\frac{\pi}{4}} = 2 \cdot \frac{\pi}{4}\sin(\frac{\pi}{4}) + 2\cos(\frac{\pi}{4}) - (2 \cdot 0\sin(0) + 2\cos(0)) = \frac{\pi}{2} \cdot \frac{1}{\sqrt{2}} + 2 \cdot \frac{1}{\sqrt{2}} - 2 = \frac{\pi}{2\sqrt{2}} + \sqrt{2} - 2$. We have used $\sin(\frac{\pi}{4}) = \cos(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$ and $\cos(0) = 1$.