Quiz 1 Friday, August 30

1 Find the antiderivative of $\frac{2}{x^3} - 2\sin(\frac{x}{2}) + \frac{\sec^2(2x)}{3}$

Solution.

First, we can simplify the expression to get
$\frac{2}{x^3} - 2\sin(\frac{x}{2}) + \frac{\sec^2(2x)}{3} = 2x^{-3} - 2\sin(\frac{x}{2}) + \frac{1}{3}\sec^2(2x)$.

Method 1:

Note that the antiderivative of x^{-3} is $\frac{x^{-3+1}}{-3+1} + c = \frac{x^{-2}}{-2} + c$, the antiderivative of $\sin(\frac{x}{2})$ is $-\frac{\cos(\frac{x}{2})}{\frac{1}{2}} + c = -2\cos(\frac{x}{2}) + c$ and the antiderivative of $\sec^2(2x)$ is $\frac{\tan(2x)}{2} + c$.

So the antiderivative of $\frac{2}{x^3} - 2\sin(\frac{x}{2}) + \frac{\sec^2(2x)}{3}$ is
$2 \cdot \frac{x^{-2}}{-2} - 2 \cdot (-2\cos(\frac{x}{2})) + \frac{1}{3} \cdot \frac{\tan(2x)}{2} + c$
which can be simplified as

$$-x^{-2} + 4\cos(\frac{x}{2})) + \frac{1}{6}\tan(2x) + c.$$

One can verify the answer by taking the derivative of $-x^{-2}+4\cos(\frac{x}{2}))+\frac{1}{6}\tan(2x) + c$ to get

$$\begin{aligned}
&\frac{d}{dx}\Big(-x^{-2} + 4\cos(\frac{x}{2})) + \frac{1}{6}\tan(2x) + c\Big) \\
&= - \cdot (-2)x^{-3} + 4 \cdot (-\cos(\frac{x}{2}) \cdot \frac{1}{2}) + \frac{1}{6}\sec^2(2x) \cdot 2 + 0 \\
&= 2x^{-3} - 2\cos(\frac{x}{2}) + \frac{1}{3}\sec^2(2x).
\end{aligned}$$

Method 2: We can use indefinite integral to find the antiderivative of a function.

Since $\frac{2}{x^3} - 2\sin(\frac{x}{2}) + \frac{\sec^2(2x)}{3} = 2x^{-3} - 2\sin(\frac{x}{2}) + \frac{1}{3}\sec^2(2x)$, we have

$$\begin{aligned}
&\int \Big(\frac{2}{x^3} - 2\sin(\frac{x}{2}) + \frac{\sec^2(2x)}{3}\Big)dx \\
&= \int \Big(2x^{-3} - 2\sin(\frac{x}{2}) + \frac{1}{3}\sec^2(2x))dx \\
&= 2\int x^{-3}dx - 2\int \sin(\frac{x}{2})dx + \frac{1}{3}\int \sec^2(2x)dx \\
&= 2 \cdot \frac{x^{-2}}{-2} - 2 \cdot (-\frac{\cos(\frac{x}{2})}{\frac{1}{2}}) + \frac{1}{3} \cdot \frac{\tan(2x)}{2} + c \\
&= -x^{-2} + 4\cos(\frac{x}{2})) + \frac{1}{6}\tan(2x) + c
\end{aligned}$$

So the antiderivative of $\frac{2}{x^3} - 2\sin(\frac{x}{2}) + \frac{\sec^2(2x)}{3}$ is
$-x^{-2} + 4\cos(\frac{x}{2})) + \frac{1}{6}\tan(2x) + c$.