Solution to Quiz 3

1

$$\text{What is } \frac{d}{dx}\int_3^x (\sec(2u) - u^2 e^{u^2})du \ \ ?$$

Solution: Recall that the fundamental theorem of calculus (Part I) implies that $\frac{d}{dx}\int_a^x f(u)du = f(x)$.

Now $f(u) = \sec(2u) - u^2 e^{u^2}$. So $f(x) = \sec(2x) - x^2 e^{x^2}$ and $\frac{d}{dx}\int_3^x (\sec(2u) - u^2 e^{u^2})du = f(x) = \sec(2x) - x^2 e^{x^2}$.

2 Evaluate the following definite integral if possible:

$$\int_1^4 \frac{-x^2+2}{\sqrt{x}}dx$$

Solution: First we simplify
$\frac{-x^2+2}{\sqrt{x}} = \frac{-x^2+2}{x^{\frac{1}{2}}} = -\frac{x^2}{x^{\frac{1}{2}}} + \frac{2}{x^{\frac{1}{2}}} = -x^{2-\frac{1}{2}} + 2x^{-\frac{1}{2}} = -x^{\frac{3}{2}} + 2x^{-\frac{1}{2}}$.

So $\int \frac{-x^2+2}{\sqrt{x}}dx = \int(-x^{\frac{3}{2}} + 2x^{-\frac{1}{2}})dx = -\frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 2\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C = -\frac{2}{5}x^{\frac{5}{2}} + 4x^{\frac{1}{2}} + C$.
This implies that
$\int_1^4 \frac{-x^2+2}{\sqrt{x}}dx = -\frac{2}{5}x^{\frac{5}{2}} + 4x^{\frac{1}{2}}\Big|_1^4 = -\frac{2}{5}\cdot 4^{\frac{5}{2}} + 4\cdot 4^{\frac{1}{2}} - \Big(-\frac{2}{5}\cdot 1^{\frac{5}{2}} + 4\cdot 1^{\frac{1}{2}}\Big)$

Note that $4^{\frac{1}{2}} = \sqrt{4} = 2$, $4^{\frac{5}{2}} = (4^{\frac{1}{2}})^5 = 2^5 = 2*2*2*2*2 = 32$, $1^{\frac{5}{2}} = 1$ and $1^{\frac{1}{2}} = 1$.

So $-\frac{2}{5}\cdot 4^{\frac{5}{2}} + 4\cdot 4^{\frac{1}{2}} - \Big(-\frac{2}{5}\cdot 1^{\frac{5}{2}} + 4\cdot 1^{\frac{1}{2}}\Big) = -\frac{2}{5}\cdot 32 + 4\cdot 2 - \Big(-\frac{2}{5} + 4\Big) = -\frac{64}{5} + 8 + \frac{2}{5} - 4 = -\frac{62}{5} + 4 = \frac{-62+20}{5} = -\frac{42}{5}$.

Thus $\int_1^4 \frac{-x^2+2}{\sqrt{x}}dx = -\frac{42}{5}$.