Solution to Quiz 5

- (30 pts)

$$\int \frac{2x^3 - x}{x^4 - x^2 + 1} dx$$

Solution: Let $u = x^4 - x^2 + 1$. Then $du = (4x^3 - 2x)dx$ and $\frac{du}{2} = (2x^2 - x)dx$. So $\int \frac{2x^3-x}{x^4-x^2+1} dx = \int \frac{1}{u}\frac{du}{2} = \frac{1}{2}\int \frac{1}{u} du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^4 - x^2 + 1| + C$.

- (30 pts)

$$\int \sin(x) e^{\cos(x)} dx$$

Solution: Let $u = \cos(x)$. Then $du = -\sin(x)dx$. So $\int \sin(x)e^{\cos(x)}dx = \int e^u(-du) = -\int e^u du = -e^u + C = e^{\cos(x)} + C$.

- (40 pts)

$$\int x^3 \ln x dx$$

Solution: Let $u = \ln(x)$ and $dv = x^3 dx$. Then $du = \frac{1}{x}dx$ and $v = \int x^3 dx = \frac{x^4}{4}$. So $\int x^3 \ln x dx = \int \ln(x) \cdot x^3 dx = \ln(x) \cdot \frac{x^4}{4} - \int \frac{x^4}{4} \cdot \frac{1}{x} dx = \frac{x^4 \ln(x)}{4} - \int \frac{x^3}{4} dx = \frac{x^4 \ln(x)}{4} - \frac{x^4}{16} + C$.

- (10 pts)

$$\text{Bonus problem} \int \frac{x^5}{\sqrt{x^2+1}}$$

Solution: Let $u = x^2 + 1$. Then $du = 2xdx$, $\frac{du}{2} = xdx$ and $x^2 = u - 1$. So $\int \frac{x^5 dx}{\sqrt{x^2+1}} = \int \frac{(x^2)^2 x dx}{\sqrt{x^2+1}} = \int \frac{(u-1)^2}{\sqrt{u}} \frac{du}{2} = \frac{1}{2}\int \frac{(u^2-2u+1)}{\sqrt{u}} du = \frac{1}{2}\int (u^{\frac{3}{2}} - 2u^{\frac{1}{2}} + u^{-\frac{1}{2}}) du = \int (\frac{1}{2}u^{\frac{3}{2}} - u^{\frac{1}{2}} + \frac{1}{2}u^{-\frac{1}{2}}) du = \frac{1}{2} \cdot \frac{2}{5} \cdot u^{\frac{5}{2}} - \frac{1}{2} \cdot \frac{2}{3} u^{\frac{3}{2}} + \frac{1}{2} \cdot 2u^{\frac{1}{2}} + C = \frac{1}{5} \cdot u^{\frac{5}{2}} - \frac{1}{3}u^{\frac{3}{2}} + u^{\frac{1}{2}} + C = \frac{1}{5}(x^2+1)^{\frac{5}{2}} - \frac{1}{3}(x^2+1)^{\frac{3}{2}} + (x^2+1)^{\frac{1}{2}} + C$.