Review Problem for Midterm #2

Midterm II: Monday, Octo.28. Topics: 7.1-7.4, 8.1-8.2 and 12.1

Office Hours before the exam: M, W 2-4pm and F 2-3 pm. Email me to make appointment if these times are not good for you.

No calculator is allowed in the exam. You should know how to solve these problems without a calculator.

1. Evaluate the following indefinite integrals:

1. $\int \frac{x+1}{(x^2+2x+10)^4}dx$
2. $\int xe^{-3x}dx$
3. $\int x\sin(3x)dx$
4. $\int_0^3 x\sqrt{x+1}dx.$
5. $\int \frac{dx}{1+\sqrt{x}}$
6. $\int \frac{1}{\sqrt{x}(1+\sqrt{x})}dx$
7. $\int \sin x\sqrt{\cos x}dx$
8. $\int \cos^5 xdx$
9. $\int_0^{\frac{\pi}{3}} \sin x\sec^2 xdx$
10. $\int x\tan^{-1}(x)dx$
11. $\int \frac{x^3}{(1+x^2)^5}dx$
12. $\int \sec^4 x\tan^3 xdx$
13. $\int e^{ax}\sin(bx)dx$
14. $\int e^{ax}\cos(bx)dx$
15. $\int \frac{dx}{e^x+1}$
16. $\int \frac{\ln x}{x^2}dx$
17. $\int x^2\ln xdx$
18. $\int \frac{\ln x}{x}dx$
19. $\int \sqrt{x}\sin(\sqrt{x})dx$
20. $\int \frac{\sin(\sqrt{x})}{\sqrt{x}}dx$
21. $\int \frac{\sin^{-1}(\sqrt{x})}{\sqrt{x}}dx$
22. $\int x\sec^2(x)dx$
23. $\int \arcsin(2x)dx$
24. $\int \tan x\ln(\cos x)dx$
25. $\int \ln(x^2+1)dx$
26. $\int x^3\sqrt{1+x^2}dx$
27. $\int \frac{x^2+10x+12}{x^3+8x^2+12x}dx$
28. $\int \frac{e^{4t}}{(e^{2t}-1)^3}dt$
29. $\int \frac{x^2}{x^4-1}dx$
30. $\int \frac{x^2}{(x+2)^{10}}dx$
31. $\int \frac{2x-6}{x^2+5x+13}dx$
32. $\int \frac{x^3-1}{x^3+x}dx$
33. $\int \frac{x+1}{x^3-x^2}dx$

2. Determine whether each integral is convergent or divergent. If the integral is convergent, compute its value.

a. $\int_1^\infty \frac{1}{x^{\frac{1}{3}}}dx$
b. $\int_1^\infty \frac{1}{x^{\frac{5}{4}}}dx$
c. $\int_0^\infty \frac{x^2}{x^3+1}dx$
d. $\int_e^\infty \frac{1}{x(\ln x)^3}dx$
e. $\int_{-\infty}^\infty x^3dx$
f. $\int_{-\infty}^\infty x^2e^{-x^3}dx$

3. Solve the differential equation $\frac{dy}{dx} = y^2 - 4y + 3$ with $y(0) = 3$.

4. Suppose that $\frac{dy}{dt} = -y^2(y-3)(y-5)$
- **(a)** Determine the equilibria of this differential equation.
- **(b)** Graph $\frac{dy}{dt}$ as a function of y, and use your graph to discuss the stability of the equilibria.
- **(c)** What can you say about the solution $\lim_{t\to\infty} y(t)$ if $y(0) = 1$ or $y(0) = 4$?

5. Suppose that $\frac{dy}{dx} = g(y)$ and the graph of $\frac{dy}{dt}$ as a function of y is given by the figure above

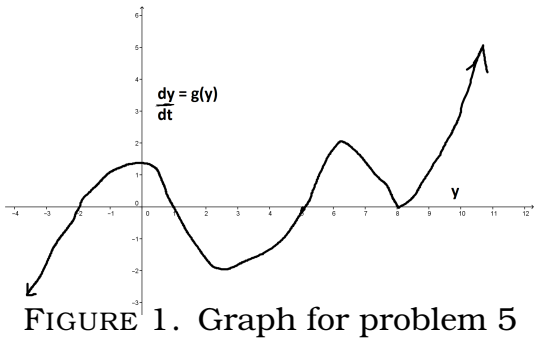

FIGURE 1. Graph for problem 5

- **(a)** Determine the equilibria of this differential equation.
- **(b)** Use the graph to discuss the stability of the equilibria.
- **(c)** What can you say about $\lim_{t\to\infty} y(t)$ if $y(0) = 3$ or $y(0) = 6$?

6. A standard deck contains 52 different cards. In how many ways can you select 7 cards from the deck?

7. Suppose you want to plant a flower bed with 3 different plants. You can choose from among 5 plants How many different choices do you have?

8. A committee of 2 people must be chosen from a group of 4. The committee consists of a president, a vice president and a treasure. How many committees can be selected?

9. An amino acid is encoded by triplet nucleotides (three nucleotides). How many different amino acids are possible if there are 4 different nucleotides that can be chosen for a triple?

10. You have just enough time to play 3 songs out of 5 from your favorite CD. In how many ways can you program your CD player to play the 3 songs?

11. Suppose that you want to investigate the effects of leaf damage on the performance of drought-stressed plants. You plan to use 5 levels

of leaf damage and 3 different watering protocol, you plan to to have 4 replicates. What is the total number of replicates?

12. Ten children are divided up into three groups, of 2, 3 and 3 children, respectively. In how many ways can this be done if the order within each group is not important?