PROBLEM H-653, THE FIBONACCI QUARTERLY, FEBRUARY, 2007

OVIDIU FURDUI

H-653. Let $n \geq 3$ be a natural number. Prove the following formula

$$\sum_{k=1}^{\infty} \frac{H_k}{k^2(k+1)(k+2)\cdots(k+n)} = \frac{2}{n!}\left(\zeta(3) - \frac{\pi^2}{8} + \frac{1}{4}\right) - \frac{1}{n!}\sum_{k=3}^{n} \frac{1}{k}\left(\frac{\pi^2}{6} - \sum_{j=1}^{k-1} \frac{1}{j^2}\right),$$

where $H_k = \sum_{j=1}^{k} 1/j$ is the k^{th} **harmonic** number and $\zeta(3) = \sum_{j=1}^{\infty} 1/j^3$ is the celebrated Apery's constant.

Department of Mathematics, The University of Toledo, Toledo, OH 43613

E-mail address: Ovidiu.Furdui@utoledo.edu, ofurdui@yahoo.com

Date: January 30, 2008.