1. (15 pts) (Problem 1 from sec 2.4) We can rewrite \((t - 3)y' + \ln(t)y = 2t\) as \(y' + \frac{\ln(t)}{t-3}y = \frac{2t}{t-3}\). We know that \(p(t) = \frac{\ln(t)}{t-3}\) is continuous for \(t \in (0, 3) \cup (3, \infty)\) and \(g(t) = \frac{2t}{(t-3)}\) is continuous for \(t \in (-\infty, 0) \cup (0, 3) \cup (3, \infty)\).

So \(\frac{\ln(t)}{t-3}\) and \(\frac{2t}{(t-3)}\) are continuous for \(t \in (0, 3) \cup (3, \infty)\). The initial value is \(y(1) = 2\) and \(1 \in (0, 3)\). The solution exists if \(t \in (0, 3)\).

2. (15 pts) (Problem 4 from sec 2.4) We can rewrite \((4 - t^2)y' + 2ty = 3t^2\) as \(y' + \frac{2t}{4-t^2}y = \frac{3t^2}{4-t^2}\). We know that \(p(t) = \frac{2t}{4-t^2}\) is continuous if \(t \in (-\infty, -2) \cup (-2, 2) \cup (2, \infty)\) and \(g(t) = \frac{3t^2}{(4-t^2)}\) is continuous for \(t(-\infty, -2) \cup (-2, 2) \cup (2, \infty)\). So both \(\frac{2t}{4-t^2}\) and \((-\infty, -2) \cup (-2, 2) \cup (2, \infty)\) are continuous for \(t \in (-\infty, -2) \cup (-2, 2) \cup (2, \infty)\). The initial value is \(y(-3) = 2\) and \(-3 \in (-\infty, -2)\). The solution of this problem exists if \(t \in (-\infty, -2)\).

3. (15 pts) (Problem 5 from sec 2.4) The initial value is \(y(1) = -3\) and \(1 \in (-2, 2)\). The solution of this problem exists if \(t \in (-2, 2)\).

4. (15 pts) (Problem 7 from sec 2.4) \(y'(t) = \frac{t-y}{2t+5y}\).

Let \(f(t, y) = \frac{t-y}{2t+5y}\). Then \(\frac{\partial f}{\partial y} = -\frac{(2t+5y)-(t-y)5}{(2t+5y)^2}\). So both \(f(t, y)\) and \(\frac{\partial f}{\partial y}\) are continuous in the region \(\{(t, y)|2t + 5y \neq 0\}\).

5. (15 pts) (Problem 8 from sec 2.4) \(y'(t) = (1 - t^2 - y^2)^\frac{1}{2}\).

Let \(f(t, y) = (1 - t^2 - y^2)^\frac{1}{2}\). Then \(\frac{\partial f}{\partial y} = \frac{-y}{(1-t^2-y^2)^\frac{3}{2}}\). So both \(f(t, y)\) and \(\frac{\partial f}{\partial y}\) are continuous in the region \(\{(t, y)|1 - t^2 - y^2 > 0\}\).

6. (25 pts) (Problem 28 from sec 2.4) Rewrite the equation \(t^2y' + 2ty - y^3 = 0\) as \(y' + \frac{2}{t}y - \frac{y^3}{t^2} = 0\).

Let \(v = y^{-2}\). Then \(\frac{dv}{dt} = -2y^{-3}\frac{dy}{dt} = -2y^{-3}(-\frac{2}{t}y + \frac{y^3}{t^2}) = \frac{2}{t}y^{-2} + \frac{2}{t^2} = \frac{4}{t}v + \frac{2}{t^2}\).

So \(\frac{dv}{dt} = -\frac{4}{t^2}v\). The integrating factor is \(\mu(t) = e^{\int -\frac{4}{t^2}dt} = e^{-4\ln t} = t^{-4}\).

The solution of \(\frac{dv}{dt} = -\frac{4}{t^2}v\) is \(v(t) = \int \frac{\mu(t)}{\mu(t)} \frac{dv}{dt} = \int \frac{\mu(t)}{\mu(t)} \frac{-4v}{t^4}dt = \int -\frac{4v}{t^4}dt = \frac{2v^2}{t^2} + C = -\frac{2v^2}{t^2} + Ct^4\).

Recall that \(v = y^{-2}\). We have \(y^2 = \frac{1}{v}\) and \(y = \pm \sqrt{\frac{1}{v}} = \pm \sqrt{-\frac{2v^{-2}}{t^2} + Ct^4}\).